

Theoretical analyses for the relationship between the performance of quadrant photodetector and the size of incident light spot

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ABSTRACT

Quadrant photodetector is one of the most popular detection devices for tip/tilt sensing. The measurement range and detection sensitivity, depending on the size of light spot incident on the quadrant photodetector, are theoretically analyzed and discussed in the application cases of the uniform irradiance distribution and of Gaussian irradiance distribution of the incident light spot. According to the theoretical results, the larger the size of the light spot is, the greater the measurement range of the quadrant photodetector, and the smaller the detection sensitivity of the quadrant photodetector.

Keywords: quadrant photodetector, sensitivity, measurement range, adaptive optics

1. INTRODUCTION

A quadrant photodetector (QPD) is widely used for position sensing. QPD offers a very high resolution of position and angular detection in a wide dynamic range. Moreover, QPD is also noise tolerant, quite simple, highly sensitive and robust. Hence the QPD is applied in the beam alignment system¹, displacement measurement², tracking system³, optical tweezers system^{4,5}, laser guidance⁶ and adaptive optics⁷.

The characteristics of position sensing for QPD have been reported in several references⁸⁻¹⁰. However, in adaptive optics QPD is used for tip/tilt sensing, and the output and the detection sensitivity of QPD are related to the incident angle of the light beam. To achieve a high performance of the QPD in adaptive optics, the relationships between the characteristics of QPD and the incident angle are analyzed. Moreover, the influence of the location of the QPD in the adaptive optics system on the measurement range and sensitivity is also studied. The paper is structured as following. In section 2 the principle of QPD is presented. Then the measurement range and sensitivity of QPD for adaptive optics system are given. Sections 3 and 4 show the characteristics and detection sensitivity of QPD in the cases of the uniform and Gaussian irradiance distributions of the light spots incident on the QPD, respectively. In section 5, an experimental system is designed and implemented to test the analytical results in section 4, and the experimental results are discussed. Finally the conclusions are given in section 6.

2. PRINCIPLE OF QPD

2.1 QPD

As shown in figure 1, there are four the independent same shape and performance zones (I, II, III and IV) of the QPD surface. So the light spot incident on the QPD is divided into four parts A, B, C and D by zones I, II, III and IV. The powers in these zones are P_1 , P_2 , P_3 and P_4 , respectively. The output aberration of the QPD in the horizontal direction, x_{diff} , can be expressed by the difference between the powers in the left zones and those in the right zones. The

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output aberration of the QPD in the vertical direction, y_{diff} , is similar to x_{diff} . x_{diff} and y_{diff} can be expressed as following:

$$\begin{cases} x_{diff} = (P_2 + P_3) - (P_1 + P_4) \\ y_{diff} = (P_1 + P_2) - (P_3 + P_4) \end{cases} \quad (1)$$

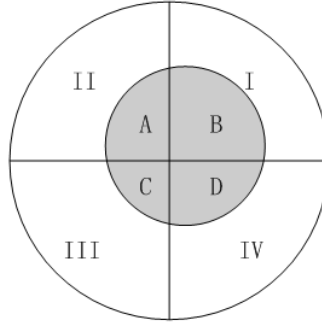


Figure 1. The diagram of the QPD surface.

To overcome the problem of the output aberration of QPD dependent on the incident intensities of the incident light beam, output aberration of QPD is normalized using the entire power incident on the QPD as the following:

$$\begin{cases} x_{diff} = \frac{(P_2 + P_3) - (P_1 + P_4)}{P_1 + P_2 + P_3 + P_4} \\ y_{diff} = \frac{(P_1 + P_2) - (P_3 + P_4)}{P_1 + P_2 + P_3 + P_4} \end{cases} \quad (2)$$

As the voltages V_1 , V_2 , V_3 and V_4 in the four zones are proportional to the powers P_1 , P_2 , P_3 and P_4 , respectively, for convenience, x_{diff} and y_{diff} can be expressed by voltages instead of powers:

$$\begin{cases} x_{diff} = \frac{(V_2 + V_3) - (V_1 + V_4)}{V_1 + V_2 + V_3 + V_4} \\ y_{diff} = \frac{(V_1 + V_2) - (V_3 + V_4)}{V_1 + V_2 + V_3 + V_4} \end{cases} \quad (3)$$

When the center of the light spot coincides with the center of the QPD surface, there is no tip/tilt aberration. At this time, x_{diff} and y_{diff} are zeros.

In order to improve the measurement range and sensitivity of QPD, the diameter of incident light beam is generally transformed by using a lens to match the size of the QPD surface. Therefore QPD is usually placed behind a lens, and the practical location depends on the needs of the measurement range and the sensitivity for tip/tilt sensing in the adaptive optics system.

2.2 Measurement range

Figure 2 is the geometric diagram of QPD in the optical system. L is a lens, D_p the diameter of the incident light beam, d the diameter of the light spot on the QPD, f the focus length of L , and l_d the distance between the location of QPD and the focal plane of L . According to the triangle similarity principle, there is

$$\frac{l_d}{f} = \frac{d}{D_p} \quad (4)$$

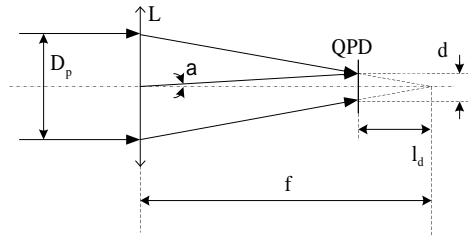


Figure 2. The geometric diagram of QPD in the optical system.

α in the figure 2 is the tilt angle between the propagation direction of the light beam and the horizontal plane. When α is zero, the aberration of tilt in the vertical direction is zero, and the center of the light spot is located on the horizontal center line of the QPD surface. And the larger α is, the larger the distance y (between the center of the light spot and the horizontal center line of the QPD surface) is. According to the geometric relationship in figure 2, y is obtained by

$$y = (f - l_d) \tan \alpha . \quad (5)$$

The order of magnitude of the tip/tilt aberration caused by the atmospheric turbulence is usually in several microradians, and α is quite small in the adaptive optics system for the ground-based telescopes. Thus y can be simplified as

$$y \gg (f - l_d) \alpha . \quad (6)$$

The measurement range of QPD in the adaptive optics system for the ground-based telescopes is not smaller than the range of the tilt angle of aberration α caused by the atmospheric turbulence. The tip/tilt aberration obeys normal distribution, its standard deviation s_{tip} ¹¹ is

$$s_{tip} = \sqrt{0.184 \frac{\lambda^3}{C^3 r_0^3} \frac{D^5}{\lambda^3} \frac{\sigma^2}{D^3}} , \quad (7)$$

where D is the telescope aperture, r_0 the Fried constant, and λ the wavelength. About 99.7% of values drawn from a normal distribution are within three standard deviations, and therefore in order to satisfy the needs for the detection of the tip/tilt loop in the adaptive optics system for the ground-based telescopes, the measurement range of QPD should be greater than $[-3s_{tip} \frac{D}{D_{tip}}, 3s_{tip} \frac{D}{D_{tip}}]$.

2.3 Sensitivity

In order to analyze the characteristics of the sensitivity of QPD, the sensitivity of QPD is defined, in this paper, as the variable quantity of the output of QPD per tilt angle. The sensitivity in the vertical direction S_y is expressed as

$$S_y = \frac{y_{diff}}{\alpha} . \quad (8)$$

In practice uniform and Gaussian intensity distributions of the light beam are often met, so the relationship between α and y_{diff} (the output characteristic of QPD) and sensitivity of QPD are theoretically analyzed for these two cases in the following.

3. UNIFORM IRRADIANCE DISTRIBUTION LIGHT

When the intensity distribution of incident light beam is uniform, the intensity distribution of light spot incident on the QPD is also uniform. Therefore the irradiance distribution $I_u(r)$ is

$$I_u(r) = \frac{P}{\pi \frac{d^2}{4}} = I_{u0}, \quad (9)$$

where r is the radius, P the entire incident light power, d the diameter of the light spot, and I_{u0} is the irradiance of the light spot. Taking the vertical direction as an example, the output characteristic and the detection sensitivity are theoretical analyzed in what follows.

3.1 Output characteristic

Because of the uniform intensity distribution, the power in each zone of the QPD can be expressed by the product of the light spot area in this zone and the irradiance I_{u0} . Using the equation (2), we obtain

$$y_{diff} = \frac{(S_1 + S_2) - (S_3 + S_4)}{S_1 + S_2 + S_3 + S_4}, \quad (10)$$

where S_1 , S_2 , S_3 and S_4 are the light spot area in the four zones I, II, III and IV, respectively. As shown in figure 3, because the light spot is symmetrical, the light spot areas of P and Q are the identical. Therefore $(S_1 + S_2) - (S_3 + S_4)$ is equal to the light spot area of T (the dotted area in the figure 3). T contains four regions of equal area, and the equation (10) can be converted into

$$y_{diff} = \frac{2}{\pi} \arcsin \frac{2y}{d} + \frac{2y}{d} \sqrt{1 - \frac{4y^2}{d^2}}. \quad (11)$$

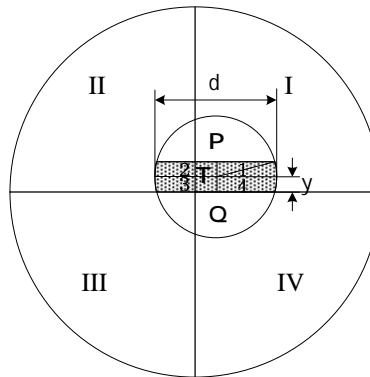


Figure 3. The light spot incident on the QPD.

The nonlinear relationship between y and y_{diff} is shown in figure 4. The abscissa is the ratio of y to $d/2$, and the ordinate y_{diff} . The larger y is, the larger y_{diff} . When y is equal to $d/2$, y_{diff} reaches the maximum value 1.

The relationship between a and y_{diff} , similar to the relationship between y and y_{diff} in figure 4, can be obtained according to the equation (6). In order to reduce the nonlinear influences for tip/tilt sensing of QPD, the QPD can be calibrated before performing real measurements. The relationship between a and y_{diff} may be stored in a ROM (Read-Only Memory). According to the output of QPD, the tilt angle is read from ROM, and the precise wavefront aberration can be obtained.

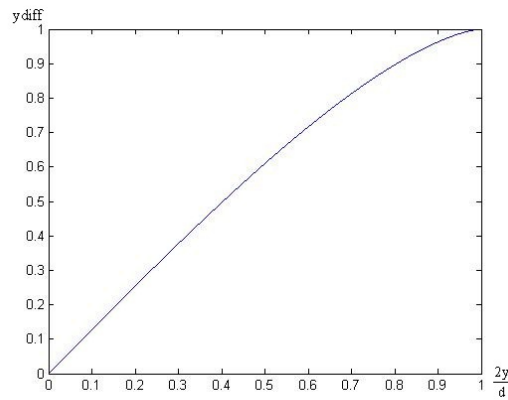


Figure 4. The relationship between y and y_{diff} (uniform irradiance distribution).

3.2 Sensitivity

Substituting the equations (6) and (11) into the equation (8), the sensitivity of QPD in the vertical direction S_y is

$$S_y \gg \frac{4(f - l_d)}{pd} \left[\frac{1}{\sqrt{1 - \left(\frac{2(f - l_d)a}{d}\right)^2}} + \sqrt{1 - \left(\frac{2(f - l_d)a}{d}\right)^2} - \frac{4(f - l_d)^2 a^2}{d^2} \frac{1}{\sqrt{1 - \left(\frac{2(f - l_d)a}{d}\right)^2}} \right]. \quad (12)$$

The equation (12) can be greatly simplified, and

$$S_y \gg \frac{8(f - l_d)}{pd} \sqrt{1 - \left(\frac{2(f - l_d)a}{d}\right)^2}. \quad (13)$$

a is a variable, and all the other parameters in the equation (13) are constant for an adaptive optics system. The larger a is, the smaller S_y .

When the center of the light spot is located on the horizontal center line of the QPD surface, the tilt aberration in vertical direction is zero, and the sensitivity S_{y0} is

$$S_{y0} \gg \frac{8(f - l_d)}{pd}. \quad (14)$$

Substituting the equation (6) into (14), S_y becomes

$$S_y \gg S_{y0} \sqrt{1 - \left(\frac{2y}{d}\right)^2}. \quad (15)$$

The relationship between y and S_y is shown in figure 5. When the wavefront aberration is zero, S_y gets the maximum value S_{y0} . When y is larger than $d/2$, S_y is 0. As y is close to $d/2$, S_y decreases sharply. When S_y is too small, the variable quantity of the output y_{diff} will be drowned by the noise. Therefore the measurement range of QPD is smaller than $[-d/2(f - l_d), d/2(f - l_d)]$.

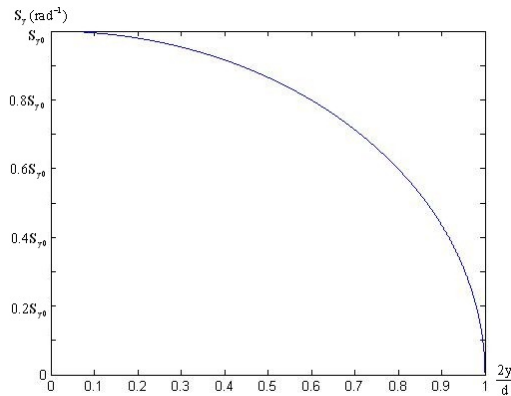


Figure 5. The relationship between y and S_y (uniform irradiance distribution).

3.3 Location of QPD in the optical system

When the location of QPD in the optical system changes, the sensitivity of QPD also varies. The relationship between S_{y0} and l_d is analyzed in this section. Substituting the equation (4) into (14), S_{y0} can be obtained as

$$S_{y0} \gg \frac{8f}{pD_p} \left(\frac{f}{l_d} - 1 \right). \quad (16)$$

In a practical adaptive optics system both f and D_p are constant. The larger l_d is the smaller S_{y0} . According to the equation (4), d is in proportion to l_d . Hence the larger d is, the smaller S_{y0} . In a like manner, the larger D_p is, the smaller S_{y0} .

In summary, the larger l_d is, the larger the effective measurement range, and the smaller the detection sensitivity. The location of QPD is determined by the needs of wavefront measurement and sensitivity in a practical adaptive optics system.

4. GAUSSIAN IRRADIANCE DISTRIBUTION LIGHT

When the incident light beam is the Gaussian beam, the intensity distribution of the light spot incident on the QPD is also Gaussian. If the irradiance at the $d/6$ is equal to the $1/e^2$ times of the maximum irradiance of the light spot, more than 99.7% energy are in the light spot. Using the above approximation, the irradiance distribution $I_G(r)$ is

$$I_G(r) \gg \frac{P}{p \frac{\pi}{4} \frac{d^2}{6}} e^{-\frac{r^2}{\frac{\pi}{4} \frac{d^2}{6}}} = \frac{36P}{p d^2} e^{-\frac{36r^2}{d^2}}. \quad (17)$$

4.1 Output characteristic

The intensity distribution of the light spot is symmetrical for the incident Gaussian light beam. The above analyses show that y_{diff} is equal to the ratio of the power in T (as shown in figure 3) to P . And the power in T is

$$P_c \gg \frac{2P}{p} \arcsin \frac{2y}{d} (1 - e^{-9}) + \frac{2P}{p} \int_0^{\arcsin \frac{2y}{d}} \frac{\pi}{4} \frac{d^2}{6} e^{-\frac{36y^2}{d^2 \sin^2 q}} \frac{1}{\sin q} dq. \quad (18)$$

Substituting the equation (18) into (2), y_{diff} is

$$y_{\text{diff}} \gg \frac{2 \arcsin \frac{2y}{d}}{p} + \frac{2}{p(1 - e^{-9})} \frac{p}{2} \frac{\arcsin \frac{2y}{d}}{d} \frac{a}{d} e^{-\frac{36y^2}{d^2 \sin^2 q}} \frac{d}{2} dq. \quad (19)$$

When the light spot obeys Gaussian intensity distribution, the relationship between y and y_{diff} is shown in figure 6. The abscissa represents the ratio of y to $d/2$, and the ordinate y_{diff} . Obviously, the relationship is nonlinear. However, as y is no more than about $d/6$, the relationship between y and y_{diff} is almost linear. When y is equal to $d/6$, y_{diff} is about 0.8. As y is greater than $d/4$, y_{diff} is close to 1. At this time, with the increase of y , y_{diff} is almost constant.

According to the equation (6), the relationship between a and y_{diff} is nonlinear. If the tilt angle a is small, y is less than $d/6$, and the output characteristic of QPD is similar to the linearity. To acquire more accurate measurement, QPD may be calibrated before performing real measurements.

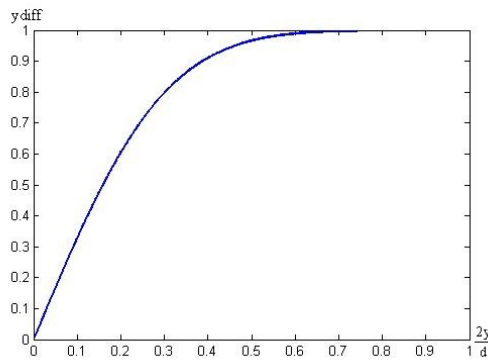


Figure 6. The relationship between y and y_{diff} (Gaussian irradiance distribution).

4.2 Sensitivity

Substituting the equation (6) and (19) into the equation (8), the sensitivity S_y is as follows:

$$S_y \gg \frac{72(f - l_d)}{pd} \frac{p}{2} \frac{\arcsin \frac{2(f - l_d)a}{d}}{d} \frac{2(f - l_d)a}{d \sin^2 q} e^{-\frac{36(f - l_d)^2 a^2}{d^2 \sin^2 q}} dq. \quad (20)$$

Assuming $S_{y_t} = \frac{72(f - l_d)}{\pi d}$, substituting the equation (6) into (20), and S_y is

$$S_y \gg S_{y_t} \frac{p}{2} \frac{\arcsin \frac{2y}{d}}{d} \frac{2y}{d \sin^2 q} e^{-\frac{36y^2}{d^2 \sin^2 q}} dq. \quad (21)$$

According to the equation (21), the relationship between y and S_y is shown in figure 7. f , d and l_d are fixed in an optical system. a is proportional to y , and the smaller a is, the larger S_y is. When the tilt angle a is zero, there's no tilt aberration, and S_y reaches the maximum. If y is larger than $3d/10$, S_y is almost zero. In this circumstance, the variation of the output of QPD y_{diff} is probably smaller than the noise, and cannot be identified. Therefore the effective measurement range of QPD is much smaller than the theoretical maximum range $[-d/2(f - l_d), d/2(f - l_d)]$.

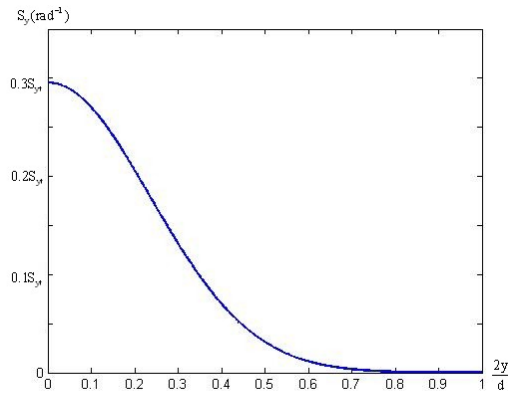


Figure 7. The relationship between y and S_y (Gaussian irradiance distribution).

4.3 Location of QPD in the optical system

The relationship between l_d and S_{y0} for a Gaussian distribution light spot is analyzed in this section, and S_{y0} is

$$S_{y0} \gg \frac{72(f - l_d)}{p d} W, \quad (22)$$

where

$$W = \lim_{a \rightarrow 0} \frac{\epsilon_p}{\epsilon_0^2} \frac{2(f - l_d)a}{d \sin^2 q} e^{-\frac{36(f - l_d)^2 a^2}{d^2 \sin^2 q}} dq. \quad (23)$$

Assuming $t_1 = \frac{2(f - l_d)a}{d}$, hence

$$W = \lim_{t_1 \rightarrow 0} \frac{\epsilon_p}{\epsilon_0^2} \frac{t_1}{\sin^2 q} e^{-\frac{9t_1^2}{\sin^2 q}} dq, \quad (24)$$

where $-\frac{9t_1^2}{\sin^2 q} < 0$, $0 < e^{-\frac{9t_1^2}{\sin^2 q}} < 1$, and then

$$0 < W < \lim_{t_1 \rightarrow 0} \frac{\epsilon_p}{\epsilon_0^2} \frac{t_1}{\sin^2 q} dq = 1. \quad (25)$$

Substituting the equation (4) into (22), S_{y0} is simplified as

$$S_{y0} \gg \frac{72f}{p D_p} \left(\frac{f}{l_d} - 1 \right) W. \quad (26)$$

In an adaptive optics system, f and D_p are constant, hence W is also constant. According to the equation (26), the larger l_d is, the larger d , and the smaller S_{y0} . In summary, the larger l_d is, the larger the effective measurement range, and the smaller the detection sensitivity.

5. EXPERIMENTS AND RESULTS

To verify the theoretical analyses of the characteristics of QPD, a series of experiments have been performed. The experimental setup is shown in figure 8. The Gaussian beam from the light source is collimated by the lens L1. The collimated beam is reflected by the fast steering mirror (FSM), and converged by the lens L2 on the QPD. FSM is controlled to change the incident angle in the horizontal and vertical direction. The characteristics of QPD in the horizon direction is similar to those in the vertical direction, hence the performance of QPD are only tested in the vertical direction.

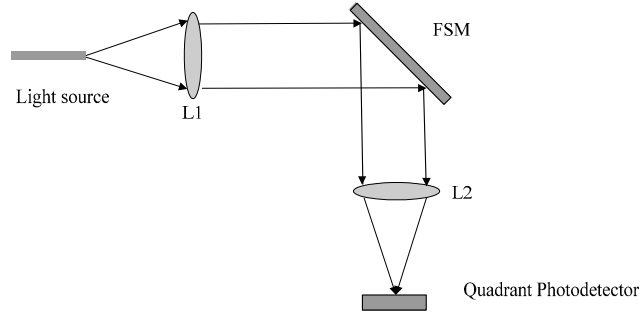


Figure 8. The schematic of the experimental system.

5.1 Output characteristic

In the experiment, D_p is 24 mm, the focus length of L2 f 200 mm, l_d 12.5 mm, and d 1.5mm. According to the equation (6), when the tilt angle of FSM is 2 mrad, the displacement of the center of the light spot y is 0.75 mm.

As shown in figure 9, the experimental results (blue solid curve) are similar to the theoretical results (black dash curve) based on the equation (19). As y increases from zero, y_{diff} increases rapidly. The energy of Gaussian irradiance distribution is concentrated in the central region of the light spot. Hence for y larger than $d/4$, as y increases, y_{diff} changes slowly.

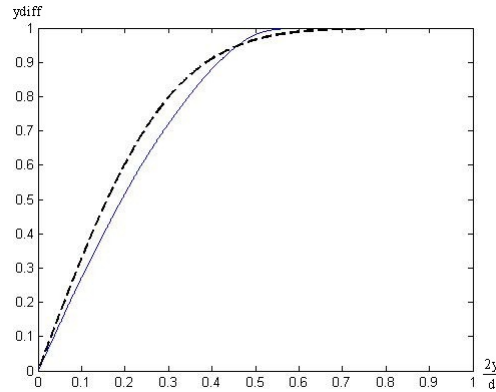


Figure 9. The experimental and theoretical results of the relationship between y and y_{diff} .

5.2 Sensitivity

The sensitivity of QPD is calculated by the Newton's difference quotient. As show in figure 10, the experimental results of the relationship between y and y_{diff} are represented by blue hexagons, and the calculated results of the relationship based on the equation (21) are represented by a black dash curve. The smaller y is, the greater S_y .

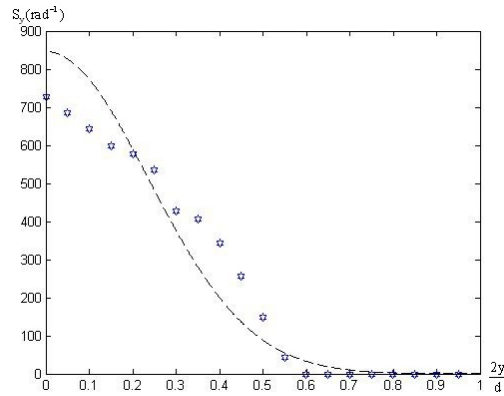


Figure 10. The experimental and theoretical results of relationship between y and S_y .

5.3 Location of QPD in the optical system

The QPD is placed on a precise micro-motion platform, and then l_d can be changed by in a magnitude order of 0.1 mm. S_{y0} at different locations is measured. The experimental results are shown in Table 1.

Table 1. The experimental results of relationship between l_d and S_{y0} .

l_d (mm)	d (mm)	S_{y0} (rad ⁻¹)
12.5	1.50	727
14.5	1.74	427
16.5	1.98	336
18.5	2.22	305
20.0	2.40	244

As shown in Table 1, the greater l_d is, the greater d , and the smaller S_{y0} , which is in good agreement with the theoretical results in section 4.3.

Based on the experimental results above, the output characteristic and the sensitivity are determined by the diameter of the light spot, which is determined by the distance from QPD to the focal plane of the lens. In order to improve the efficiency of QPD, the location of QPD must be properly chosen.

6. CONCLUSIONS

The measurement range and detection sensitivity of QPD in adaptive optics have been given. In the cases of the uniform and Gaussian irradiance distribution of light spot, the characteristics output and the detection sensitivity of the QPD are theoretically analyzed. The further the location of the QPD from the focal plane of the lens is, the larger the effective measurement range, and the smaller the detection sensitivity. Finally, in the case of the Gaussian irradiance distribution of the light spot, a series of experiments are performed. The experimental results are in good agreement with the theoretical analyses.

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