Optical Misalignment Sensing for Optical Aperture Synthesis Telescope by Using Phase Diversity

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ABSTRACT

An optical aperture synthesis telescope such as the LBT will suffer from phase errors unless the apertures are aligned to within a small fraction of a wavelength. A phase diversity wave-front sensor can measure phase errors brought by a misaligned aperture synthesis telescope. The Phase Diversity is formulated in the context of nonlinear programming where a metric is developed and then minimized. We here introduce a novel Genetic Algorithms (GAs), evaluate the different Zernike coefficients to obtain the wave-front error. The results of the computer simulations performed with simulated data including the effects of noise are shown for the case of random misalignment phase errors on each of three sub-telescope.

KEYWORDS: Aperture synthesis telescope; Wave-front sensors; phase diversity method; Genetic Algorithms;

1.INTRODUCTION

The trend in telescope design in recent years has favored aperture synthesis telescope systems. Examples of such systems include LBT. Aperture synthesis telescope have the potential to achieve resolutions far superior to that of a single segment while avoiding the considerable problems encountered in the fabrication of a monolithic primary mirror large enough to achieve a comparable resolution. In order to reach the full resolution potential, however, the individual sub telescope must be aligned to within a small fraction of a wavelength. Thus, the problems of alignment among sub telescope is introduced. Considerable effort has been directed toward the development of adaptive optics for sensing piston and tilt misalignments. For example, the Hartmann-Shack wave-front sensor is often used for measuring misalignments. In addition, laser interferometers and lateral-effect detectors have been used to measure piston and tilt misalignments . But, these techniques require considerable additional optical hardware that could also be subject to misalignments. In addition, the sensitivity of the Hartman sensor degrades as the objects being imaged become more extended[1].

Phase aberrations that come from misalignment may also be inferred directly from the image data. A technique known as phase diversity can also be used to infer phase aberrations from image data while accommodating extended objects or even scenes. The phase diversity concept was originally discussed by Gonsalves and is an extension of the phase-retrieval concept that uses two images, one of which contains an additional known aberrations, to determine the optical-system aberrations[2].

The presence of aberration in an optical system can be mathematically represented by Zernike polynomials[3]. For the purpose of sensing misalignment aberrations it may be necessary to estimate several coefficients in the Zernike decomposition of the wave front function. A nonlinear optimization technique such as the gradient descent or conjugate gradient method is then used to estimate the Zernike coefficients that would minimize the error metric. Carreras et al[4]

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used the conjugate gradient method for non-linear optimization, using finite different methods to calculate the gradients. But the above method is the possibility of becoming entrapped in a local minimum that is not the true solution. One way to avoid this problem is to perform the minimizations many times with different initial estimates each time. The hope is that at least one of the minimizations will lead to the true solution. So a large number of iterations have to be carried out.

In this paper we show that Genetic Algorithms (GAs) to estimate the Zernike coefficients. Genetic Algorithms (GAs) are adaptive heuristic search algorithm premised on the evolutionary ideas of natural selection and genetic. The basic concept of GAs is designed to simulate processes in natural system necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem. The advantages of such a method are self-evident, such as independent evaluation of the coefficients, and it needn't initial estimate and avoid local minimization problem.

2.STATEMENT OF PROBLEM

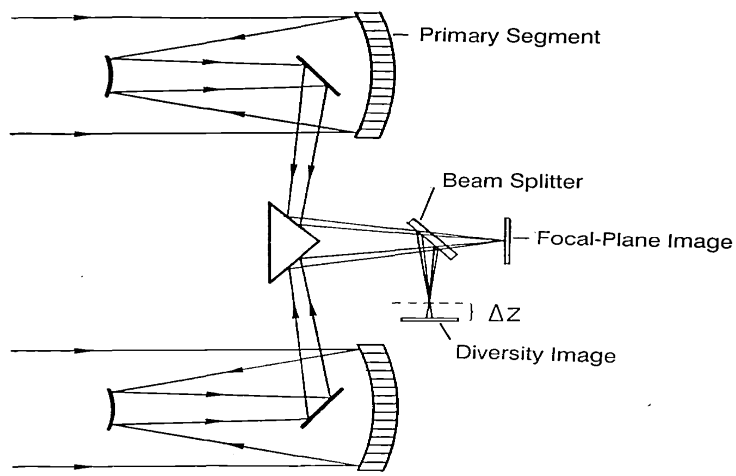


Fig1. Telescope optical system and image-collection procedure of phase diversity method

As a wave-front sensing technique, phase diversity can estimate pupil phase from pairs of simultaneously recorded focused and defocused images. From Fig.1, notice that a beam splitter has been introduced so that a second image can be collected. The second image is intentionally translated along the optical axis by a distance ΔZ so that a known amount of defocus is introduced. The goal is to infer the misalignments parameters from the two collected images.

We begin by making the simplifying assumption that the object or scene to be imaged is illuminated with spatially incoherent, quasi-monochromatic light. In addition, the imaging system is presumed to be a linear shift-invariant system, leading to the following imaging equation：

$$g(x) = f(x) * s(x) \quad (1)$$

Where *g(x)* is the given image, *f(x)* is the object to be found, *s(x)* is the point-spread function（PSF）of the system, *x* is a two-dimensional vector, and the asterisk represents a two-dimensional convolution. The PSF may have an unknown aberration associated with it that is due to system misalignment. The Fourier representation of Eq(1) is given by

$$G(u) = F(u)S(u) \quad (2)$$

The optical transfer function(OTF) of the system, $S(u)$, is found by autocorrelation of the coherent transfer function(CTF),

$$S(u) = C(u) \oplus C(u) \tag{3}$$

For a aperture synthesis telescope the CTF takes the form

$$C(u) = \sum_{n=1}^{N} P_n(u) \exp\{i\phi_n(u)\} \tag{4}$$

Where $P_n(u)$ is the binary aperture function representing the nth segment, and N is the total number of segments. It is often convenient to parameterize the unkown phase-aberration function:

$$\phi(u) = \sum_{j=1}^{J} \alpha_j \phi_j(u) \tag{5}$$

Where J coefficients in the set$\{\alpha_j\}$ serve as parameters and $\{\Phi_j\}$ is a convenient set of basis functions, such as discretized Zernike polynomials for a monolithic aperture or piston and tilt basis functions, used to represent misalignments in a phased-array system.

The method of phase diversity requires that a second image be collected. This second image is intentionally defocused. The following are the corresponding companion equations for the defocused , diverse system.

$$g_d(x) = f(x) * s_d(x) \tag{6}$$

$$G_d(u) = F(u) S_d(u) \tag{7}$$

$$S_d(u) = C_d(u) \oplus C_d(u) \tag{8}$$

where the coherent transfer function for the diverse system is the following equation.

$$C_d(u) = \sum_{n=1}^{N} P_n(u) \exp\{i[\phi_n(u) + \Delta\phi(u)]\} \tag{9}$$

Notice that the coherent transfer function for the diverse optical system has the known defocus term added to the exponential and is designated as $\Delta\Phi(u)$,the quadratic phase diversity depends on the wavelength, λ, the focal length of the system; and the distance that the diversity image is translated the optical axis, Δz :

$$\Delta\phi(u) = 2\pi\Delta w |u|^2 \tag{10}$$

where $|u|$ is the length of the vector u. For Eq(10) it is assumed that the origin of the coordinate system defining the vector u coincides with the optical axis, that is, the origin is located at the center of the three-segment array. The number of waves of quadratic diversity at the edge of the entire three sub apertures is given by

$$\Delta w = \frac{\Delta z}{8\lambda (F^{\#})^2} \tag{11}$$

where $F^{\#}$ represents the F number(focal length/diameter ratio) of the system. For optical aperture synthesis telescope , diameter ration is diameter of effective aperture.

3. ADDITIVE GAUSSIAN NOISE CASE

We now consider the case in which the noise at each detector element is modeled as an additive, independent, and identically distributed random variable with a zero-mean Gaussian probability density having a variance σ_n^2. Such a model would be appropriate, for example, if the dominant noise were thermal noise. In this case， we give the next error metric[5] :

$$L_M(\alpha) = \sum_{u \in \chi_1} \frac{\left| \sum_{j=1}^{K} D_j(u) S_j^*(u) \right|^2}{\sum_{l=1}^{K} \left| S_l(u) \right|^2} - \sum_{u \in \chi} \sum_{k=1}^{K} \left| D_k(u) \right|^2 \tag{12}$$

Remarkably, Eq(12) express a figure of merit for the correctness of the OTF estimate that is indepe- ndent of any object estimate.

Misalignment sensing can be accomplished as follows. Because the phase aberrations may be parameterized (e.g., with piston and tilt parameters), a given parameter-set estimate allows for the computation of the corresponding estimates of the conventional and diversity CTF's and subsequently the OTF's. Consequently, the error metric in Eq(16) can be computed.

4. SIMULATIONS

Computer simulations were used to test the method of optical misalignment sensing. A three sub-apertures synthesis telescope was designed to fit within a 64*64 complex array, as shown in Fig.2.

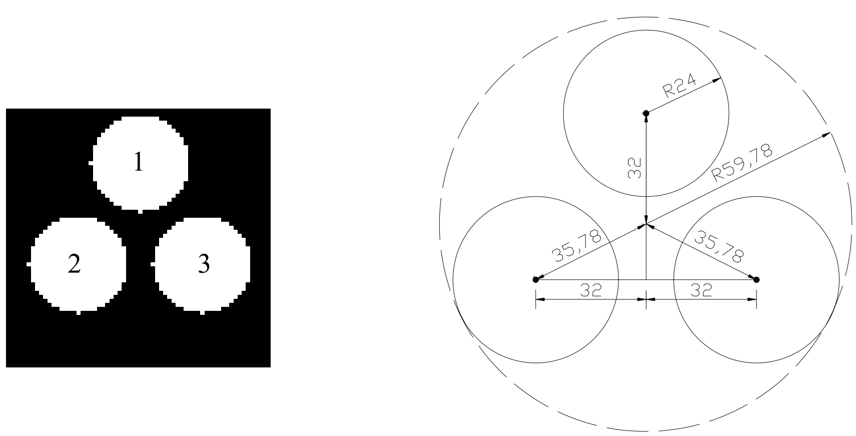


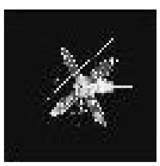
Fig.2 simulations of the aperture for a Aperture Synthesis Telescope

Various amounts of piston phase were added to each of three sub-telescope. Exit pupil diameter of sub-telescope is 480mm, f=20m, working wavelengths is 600nm.A random set of misalignment parameters of particular interest is shown in Table.1. This particular misalignment configuration, consist of only piston terms, was used as the actual configuration, or true solution, in all the simulations presented here. The corresponding OTF was created by autocorrelating the CTF, which was accomplished by embedding the CTF in a 64*64 complex array, performing a fast Fourier transform (FFT), taking the modulus squared, and applying an inverse FFT.

Table.1 misalignment configuration representing a true solution for simulations

Segment Number	Waves of Piston
1	0
2	0.84
3	0.50

A 64*64 satellite images was used to represent the true simulated object and is shown in Fig.3. The amount of diversity used here was 1.0 waves. Fig.4(a) and Fig.4(b) correspond to the focal-plan and diversity images with 1.0% additive zero-mean Gaussian white noise collected with the misaligned system[6].



(a) (b)

Fig.3 Simulated object

Fig.4. focal-plan image and diversity image collected

We here introduce Genetic Algorithms (GAs) to search piston errors[7]-[9]. Fig.5 is flowchart of Gas. Parameter of GAs as follows: Mutation rate =0.4;GenScale=[-1,-1;1,1]; FlockSize=500;the number of iterations are 30.

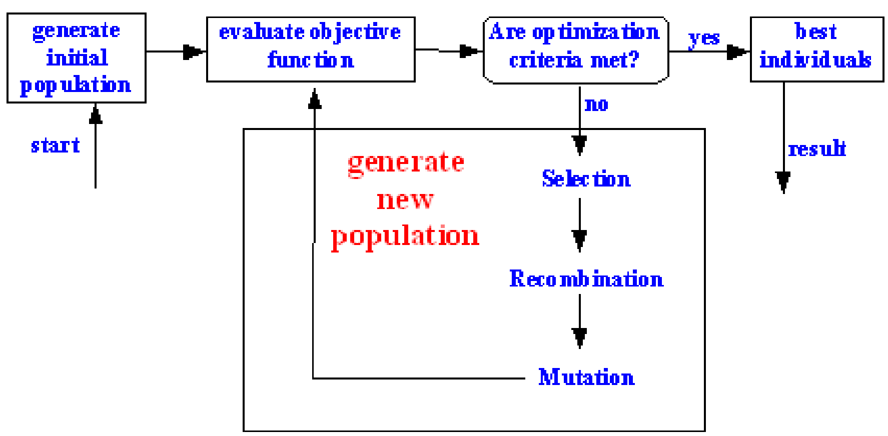


Fig.5 Flowchart of GAs

5. RESULTS

Fig.6 shows cost function varying with iteration number, it shows that the results tend to be stable when the number of iterations is 30.

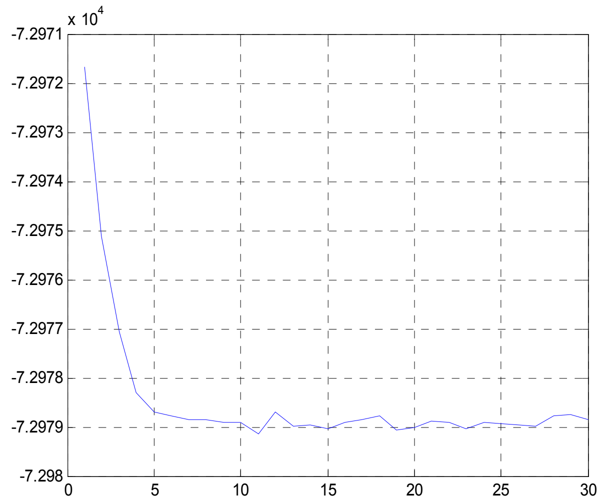


Fig.6 Cost function varying with iteration number

Table.2 shows reconstruction piston errors.

Table.2 reconstruction piston errors

Segment Number	Final Estimate(wave)	True Solution(wave)	RMSE
1	0	0	0
2	0.8577	0.84	0.0177
3	0.4607	0.50	0.0393
Average error：0.0285(wave)			

The average percentage error of the computed piston values from the given piston values was no more than 0.03waves. This error may be attributed to the effect of round-off errors and choice of parameters in GAs.

Computer simulations are presented that demonstrate the phase diversity techniques that introduce GAs successfully on a three-segment system for which only piston misalignments are allowed. Simulation result reveals that this GAs can accurately measures aberrations introduced by misalign. The advantages of GAs to evaluation of the Zernike coefficients are many. First, It avoid local minimum problem . The other hand the total number of calculation needed is less.

Our future plans include efforts to optimize GAs and improve the algorithm's accuracy to perform the Phase diversity technology using GAs estimate tip/tilt error, we have already started looking into these possibilities.

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