Error analysis of friction drive elements

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ABSTRACT

Friction drive is used in some large astronomical telescopes in recent years. Comparing to the direct drive, friction drive train consists of more buildup parts. Usually, the friction drive train consists of motor-tachometer unit, coupling, reducer, driving roller, big wheel, encoder and encoder coupling. Normally, these buildup parts will introduce somewhat errors to the drive system. Some of them are random error and some of them are systematic error. For the random error, the effective way is to estimate their contributions and try to find proper way to decrease its influence. For the systematic error, the useful way is to analyse and test them quantitively, and then feedback the error to the control system to correct them. The main task of this paper is to analyse these error sources and find out their characteristics, such as random error, systematic error and contributions. The methods or equations used in the analysis will be also presented detail in this paper.

Keywords: friction drive, drive system error, error analysis, error calculation

1. INTRODUCTION

Normally the friction drive system consists of driving motor-tachometer unit, driving roller, big wheel, control feedback device and coupling, shown as Fig.1 schematically. The main error introduced by these components to the motion accuracy is summarized in Fig.2. Among these errors, some are systematic error which can be calculated or tested with the help of certain instruments, and then compensated through control system. But, some of them can not, we call them random error. A useful way to decrease the influence of random error to motion accuracy is through careful design of structure, fine maching of some important unit, careful choice of control components and fine adjustment of the whole system. The analysis and calculation of above errors will be given in this paper, taking the experiment rotating table shown in Fig. 3 as example. The components, analysed in this paper, include motor-tachometer, encoder connector, surface quality of driving roller, alignment between driving roller and big wheel, slippage occurring in the contact surface between roller and wheel. The summarization will be given in the conclusion section.

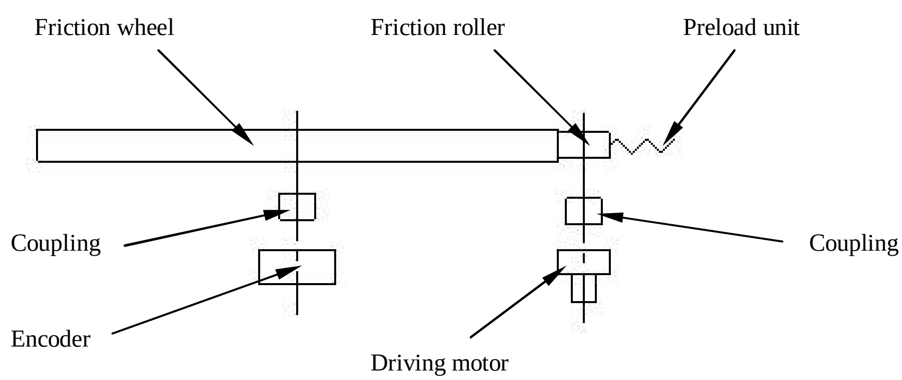

Fig.1 Scheme of friction drive unit

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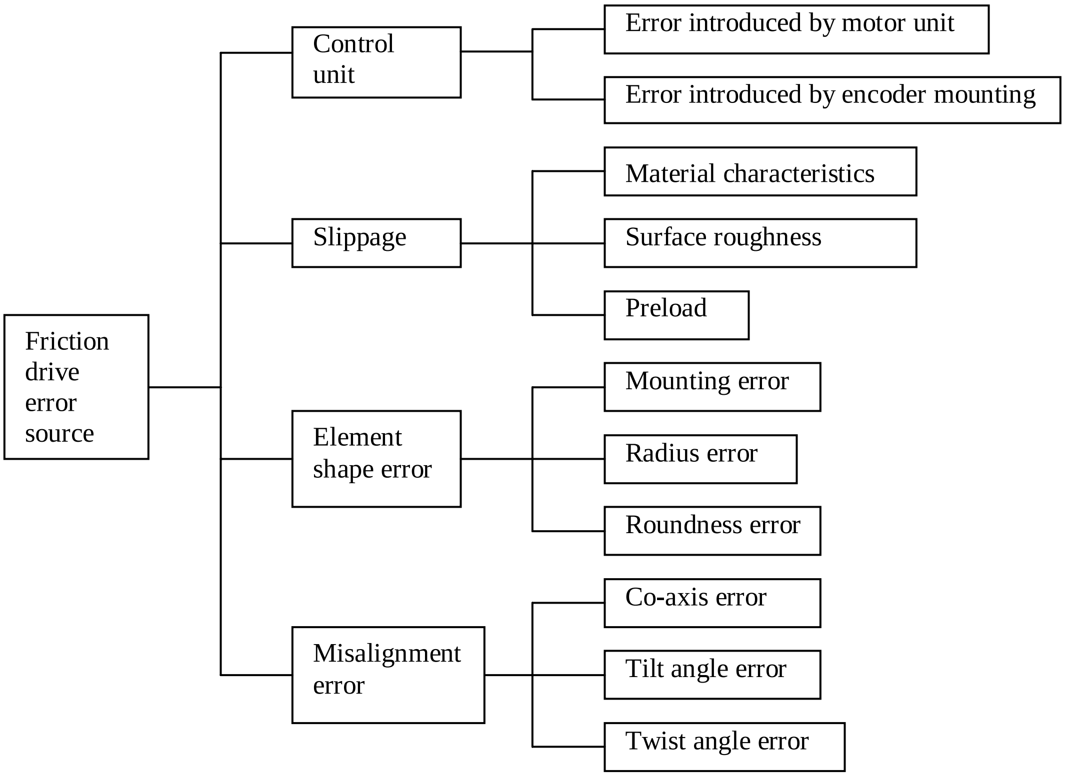

Fig.2 Friction drive error source

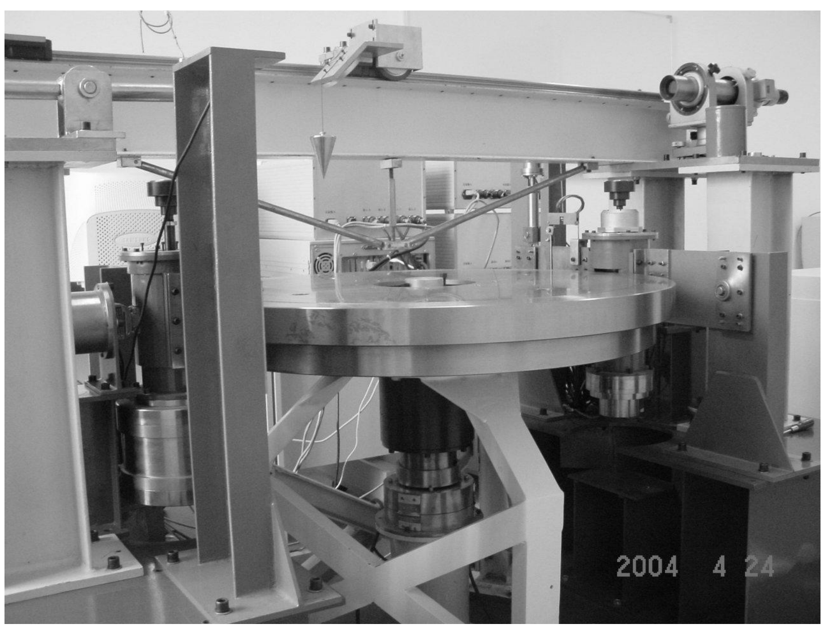

Fig.3 Friction drive test bench

2. ERROR ANALYSIS

2.1 Error introduced by motor and tachometer

Usually the driving motor and tachometer is manufactured as one integrated part. The permanent magnet motor is the power to drive the roller, so, it is the permanent magnet motor that will introduce main error to the drive accuracy. The main factor which will impact the motor motion performance is the torque fluctuation, including torque variation introduced by cogging effect, torque fluctuation introduced by friction torque inside the motor, etc. When the motor rotates in a constant velocity Ω_0, it is regarded roughly that the torque fluctuation $M_d(t)$ is only relative to the time t. The $M_d(t)$ is expressed as the following equation:

$$M_d(t) = A\sin(\omega_z t)$$

Where, $\omega_z = \Omega_0 Z$, Z is the teeth number of motor rotor. The angle velocity variation introduced by disturbance torque is given by the following equation:

$$\delta_1 = \frac{|\Omega|}{\Omega_0} = \frac{PAZ}{\sqrt{(\omega_n^2 - \omega_Z^2)^2 + 4\xi^2\omega_n^2\omega_Z^2}} \qquad (1)$$

Where, $|\Omega|$ is the angle velocity modular introduced by disturbance torque, P=1/J, $\omega_n=(K_1K_2/J)^{1/2}$, $\xi = K_TK_2/2\omega_n J$.

K_1: position loop gain, K_2: velocity loop gain, K_T: velocity feedback coefficient.

Taking the parameters of the experiment friction drive rotating table, according to the above formula, the velocity variation is about 1.13‰.

2.2 Error introduced by encoder mounting

Normally, the encoder is connected with rotating shaft using elastic coupling. Encoder is the high accuracy position feedback device for the control system, so, the main task of the coupling is to let the encoder to feedback the actual position of the test bench. For the experiment test bench, Heidenhain RON905 was chosen as the measure encoder with 0.008″ resolution and 0.05N-m friction torque introduced by bearings. In order to ensure the encoder accuracy, more attention should be paid on the following issues:

- ✓ Having enough rotation stiffness to keep the rotation accuracy;
- ✓ Having the capability to absorb the runout in other DOFs, such as tilt, radial runout, axial runout;
- ✓ No extra force applied to the encoder due to the misalignment of connected shafts;

Through the above analyses, the main errors introduced by coupling are as followings:

- ✧ Error due to the non co-axis between encoder axis and tested shaft;
- ✧ Error due to coupling elastic twist;
- ✧ Error due to the coupling rotation angle variation when it absorbs the tilt, radial runout and radial runout of the axis;

a) Non co-axis error between connected axes

There are three situations about the non co-axis error between encoder shaft and connected shaft, shown as the following Fig.4.

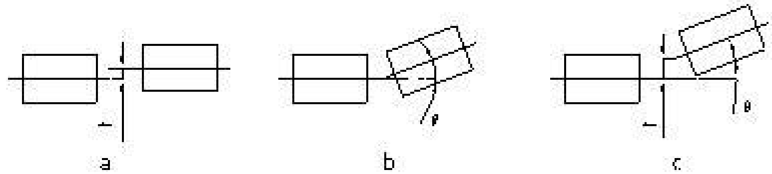

Fig. 4 Three situations about misalignment between encoder and coupling shaft. a: offset b: tilt c: offset plus tilt

For the misalignment situation between rotating table shaft and encoder axis, shown as Fig. 5, the transmission angle error can be calculated according to the following formula:

$$\Delta\phi = \phi_1 - \phi_2 = \arcsin\left(\frac{t}{r}\sin(\phi_2)\right) \qquad (2)$$

$$|\Delta\phi_{max}| = \arcsin\left(\frac{t_{max}}{r}\right) \qquad (3)$$

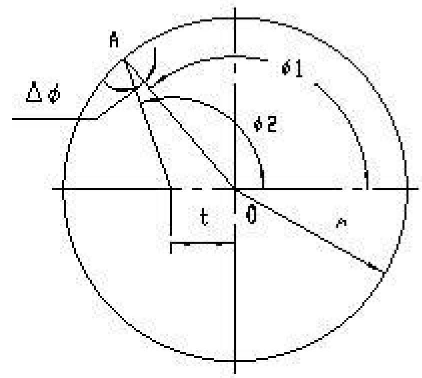

Fig.5 Misalignment between encoder axis and test shaft

Where: Φ_1: encoder rotation angle.

Φ_2 : rotation angle of rotating table shaft.

t: offset between encoder axis and rotating table shaft.

r: encoder shaft radius.

The error introduced by misalignment is the systematic error, which can be corrected by control system. But, this misalignment will cause the system unstable, such as stir. On the other hand, this misalignment will fasten the bearing wear and then impact the system accuracy.

b) Error introduced by coupling twist angle

Elastic coupling twist angle error is relative to the twist stiffness and rotation torque. It can be calculated by the following equation:

$$\theta = \frac{M}{C}$$

Where:

θ : twist angle; M: rotation torque; C: coupling twist stiffness;

The encoder selected for the rotating table is Heidenhain RON905 shown as Fig. 6. It will be connected to a 120mm diameter rotating shaft. The coupling between encoder and rotating shaft is shown as Fig.7.


Fig.6 RON905 encoder used in test bench

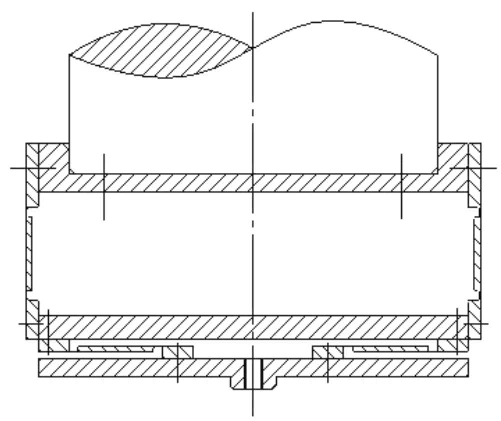
Fig.7 Coupling between encoder and test shaft

In order to meet the requirements, the coupling consists of three level plate structure, shown as Fig. 7. Top plate will be connected with shaft and will be connected with middle plate using six metal elastic sheets shown as Fig.8. The shaft radial runout and tilt will be compensated by the elastic deformation of these elastic sheets. The middle and bottom plate will be connected using four elastic sheets shown in Fig.9. Also, the axial runout and tilt will be compensated by these elastic sheets. For this rotating table, t is 0.4mm and b is 3mm. With the help of these thin grooves, the elastic sheet can compensate the tilt, radial runout and axial runout. Meanwhile, this kind of coupling has enough rotation stiffness to transmit rotation torque.

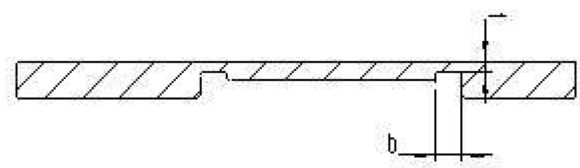

Fig.8 Elastic sheet to connect the top plate and middle plate

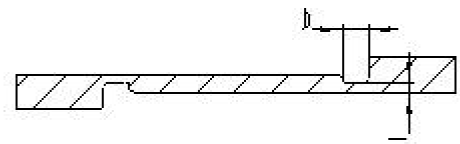

Fig.9 Elastic sheet to connect the middle plate and bottom plate

According to the elastic mechanics, the relative torsion angle between top plate and middle plate is given by the following formula:

$$\theta_{\max\ 1} = \frac{M_{\max}}{GI_p} \cdot \frac{180 \times 3600}{\pi} \cdot L \qquad (4)$$

Where:

$\theta\ max_1$ is the maximum torsion angle (″); M_{max} is the maximum torque, for RON905, M_{max}=0.05N.m; G is the shear elastic modular, for steel, G=80GP$_a$(80×10^9 N / m^2); I_p is moment of inertia; L is the torsion length.

The torsion angle between middle plate and bottom plate will be calculated using the above same method.

As Fig.7 shows that the radial runout between encoder and connected shaft will be compensated by the grooves of elastic sheet connecting top plate and middle plate. The corresponding extra radial force P_r introduced by radial runout is given by the following formula according to elastic mechanics:

$$P_r = \frac{3EI}{L^3} \cdot y_{\max} \qquad (5)$$

Where: E: elastic modulus, 220GP$_a$; I: moment of inertia. L: groove length; y_{max}: max. radial runout.

The corresponding torque introduced by P_r is as following:

$$M_r = \frac{1}{2}\mu d p_r$$

Where: μ is the friction coefficient; d is the diameter.

According to formula (4), the torsion angle introduced by radial runout can be calculated.

Meanwhile, the axial runout of two connected shafts can be compensated by grooves of elastic sheet connecting middle plate and bottom plate shown as Fig.7. With the help of same method, the torsion angle introduced by axial runout can also be calculated.

2.3 Gear ratio variation introduced by roller or wheel radius error

For a friction drive system, the gear ratio is as following:

$$i = \frac{\omega_2}{\omega_1} = \frac{r}{R} \quad \Longrightarrow \quad di = \frac{1}{R}\left(dr - i \bullet dR\right) \qquad (6)$$

After the roller and wheel was manufactured, the radius deviation △R or △r is confirmed. That is to say, the gear ratio variation is a constant which can be corrected by control system.

2.4 Error introduced by roller elliptical deviation

Theoretically, the outline of roller is round. But, due to mechanical maching error, the roller outline is not the ideal one. For example, elliptical shape is the typical maching error shown as Fig. 10. R is the radius of ideal circle. a is the semimajor axis and b is semiminor axis. The ellipse perimeter is calculated by the following equation:

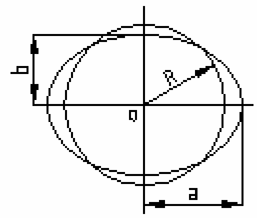

Fig.10 Ellipical shape of roller

$$L = 4\int_0^{\frac{\pi}{2}} \sqrt{a^2 \cos^2\theta + b^2 \sin^2\theta}\, d\theta$$

$$= 4a\int_0^{\frac{\pi}{2}} \sqrt{1 - \frac{a^2 - b^2}{a^2}\sin^2\theta}\, d\theta$$

$$\approx \pi\left(\frac{a+b}{2} + \sqrt{\frac{a^2+b^2}{2}}\right) \qquad (7)$$

So, the perimeter variation is $\Delta L = L - 2\pi R$

Consequently, the angle variation is

$$\Delta\theta = \theta\left(1 - \frac{2\pi R}{L}\right) \qquad (8)$$

2.5 Error introduced by roller or wheel mounting

The mounting error is schemed as Fig.11. C_1 and C_2 are the geometrical centers of roller and wheel. O_1 and O_2 are the rotation center of roller and wheel. e_1 and e_2 are the corresponding eccentric throw. At the moment of t, the wheel and roller contacts at k point which is in the line connecting c_1 and c_2. The linear velocity of wheel and roller at point k are given as:

$$v_1 = \omega_1 \cdot \overline{o_1 k} \qquad\qquad v_2 = \omega_2 \cdot \overline{o_2 k}$$

According to the friction drive requirement, the velocity component of wheel in the tangent direction of contact point k should be the same as the velocity component of roller in the tangent direction of contact point k.

$$\omega_1 \cdot \overline{o_1 k} \cdot \cos\beta_1 = \omega_2 \cdot \overline{o_2 k} \cdot \cos\beta_2$$

$$i = \frac{\omega_2}{\omega_1} = \frac{\overline{o_1 k} \cdot \cos\beta_1}{\overline{o_2 k} \cdot \cos\beta_2}$$

From the Fig.11. there are the following relations.

$$\overline{o_1 k} \cdot \cos\beta_1 = r + e_1 \cdot \cos\alpha_1 \qquad \overline{o_2 k} \cdot \cos\beta_2 = R + e_2 \cdot \cos\alpha_2$$

$$\alpha_1 = \omega_1 t + \theta \qquad\qquad \alpha_2 = \omega_2 t + \theta$$

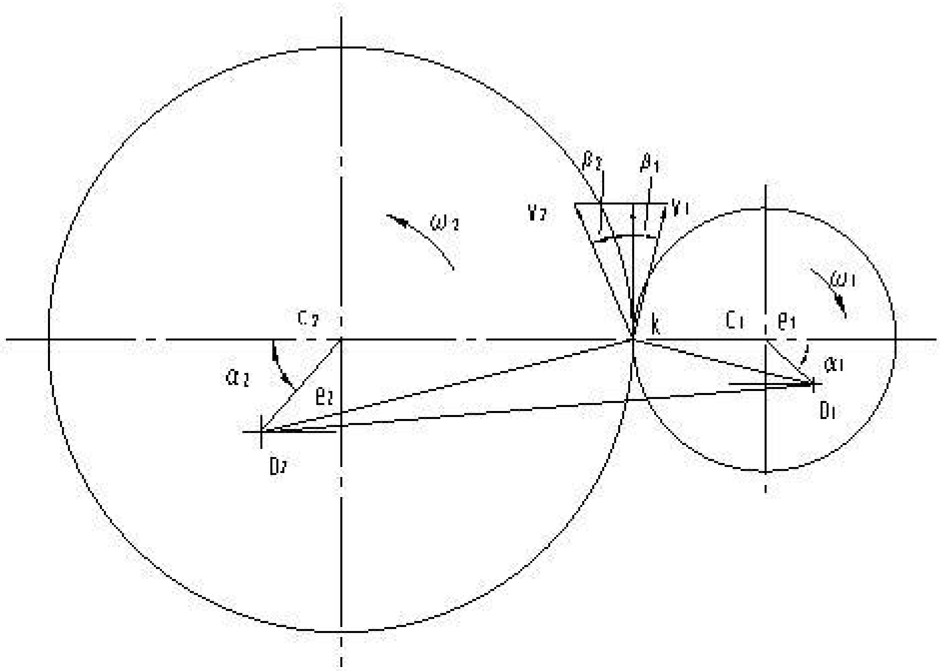

Fig.11 Scheme of mounting eccetrical of wheel and roller

Where: r is the roller radius, R is the big wheel radius, θ is the angle between O_1O_2 and C_1C_2. The θ will be the maximum value when O_1, O_2 are at the same side of C_1C_2. θ_{max} will be given as the following equation:

$$\left|\theta_{max}\right| = \frac{e_1 \sin(\alpha_1) + e_2 \sin(\alpha_2)}{r + R + e_1 \cos(\alpha_1) + e_2 \cos(\alpha_2)} \le \arcsin\left(\frac{e_1 + e_2}{r + R}\right) << 1$$

Then

$$i \approx \frac{r + e_1 \cos \omega_1 t}{R + e_2 \cos \omega_2 t} = \frac{r}{R}\left(\frac{1 + \frac{e_1}{r} \cos \omega_1 t}{1 + \frac{e_2}{R} \cos \omega_2 t}\right) \tag{9}$$

$$i \approx i_0\left(1 + \frac{e_1}{r} \cos \omega_1 t - \frac{e_2}{R} \cos \omega_2 t\right) \tag{10}$$

Where: $i_0 = \dfrac{r}{R}$

When the roller runs in constant angle velocity ω_1, the big wheel angle velocity variation is as:

$$\Delta\omega_2 = \omega_2^{'} - \omega_2 = i \cdot \omega_1 - \omega_2$$

$$= \frac{\omega_2 \cdot e_1}{r} \cos \omega_1 t - \frac{\omega_2 \cdot e_2}{R} \cos \omega_2 t$$

$$= \frac{\omega_1}{R} \left(e_1 \cos \omega_1 t - i_0 e_2 \cos \omega_2 t \right) \tag{11}$$

At moment t, the roller rotates $\theta_1 = \omega_1 t$. For big wheel, the corresponding angle is $\theta_2 = i_0 \times \theta_1 = \omega_2 t$. The rotation angle variation of wheel can be calculated through integral of equation (11) as the following:

$$\Delta\theta_2 = \frac{1}{R} \left[e_1 \sin \theta_1 - e_2 \sin(i_0 \theta_1) \right] \tag{12}$$

2.6 Error introduced by slippage between wheel and roller

For the friction drive system, there are three slippage phenomena: micro slippage, geometry slippage and macro slippage. The macro slippage is due to the overload of the system. It can be settled by adding large preload between wheel and roller. The geometry slippage is due to the mounting position error. Theoretically, the axes of roller and big wheel should be parallel to each other in spacial, that is to say, the roller should be adjusted so well that its axis of rotation is parallel to that of the surface on which it is rolling in all directions. In practical, it is hard to do so and then there are two kind of skew angle. One kind of skew angle is along the radial direction, and another one is in the tangential direction, just as Fig.12 shows. Because the roller is supported by elastic plate, the skew angle in radial direction can be eliminated under the enough preload, but it does not work for the skew angle in tangential. If the roller is misaligned in tangential, this will generate an axial motion of the roller which increases as the roller rotates, just as twist-roller friction drive. This axial motion will be resisted by the roller bearings, and will give rise to an axial back force. When this force exceeds the frictional force between the contact surfaces, an elastic slip will occur. This will induce the uneven motion and introduce sharp errors in the control system. What is more, the roller misalignment may also cause premature wear on the disks and damage to the roller, the wheel and its bearings. The relationship between axial displacement S and skew angle θ is followed as: $S = \pi \times D \times tg\theta$ (D: the diameter of roller).

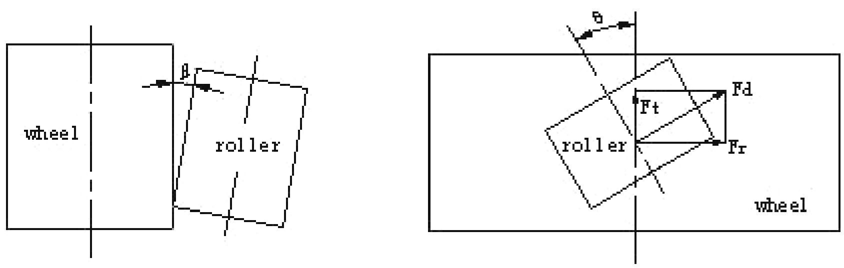

Fig.12 Schematic diagram of skew angle

From the above analysis, the way to settle the geometry slippage is to get rid of the twist angle between roller and wheel by means of a set of test and adjustment unit.

Elastic slippage is unavoidable for elastic material wheel and roller. It is usually given as:

$$\delta_3 = \mu \left(\frac{R_1 + R_2}{R_1 R_2} \right)^{\frac{1}{2}} \left(\frac{4N}{\pi B} \right)^{\frac{1}{2}} \left[\frac{2(1-\nu^2)}{E} \right]^{\frac{1}{2}} \left[1 - \left(1 - \frac{Q}{\mu N} \right)^{\frac{1}{2}} \right] \tag{13}$$

Where: R_1, R_2 are the radius of wheel and roller respectively. N is the preload applied on the contact surface. B is the contact length. ν is Poisson's ratio. μ is the friction coefficient. Q is the driving torque. Normally, the micro slippage is very small, about 2%.

3. CONCLUSION

This paper summary the error incurring factors for the friction drive system. When the position accuracy requirement reaches sub-arcsec, many factors will be considered carefully. Firstly, it is better to identify their characteristics: systematic error or random error, and then to take corresponding measures to deal with. The systematic error, introduced by the same reason or affects the results in the same direction, such as driving ratio error, mounting position error, can be detected with the help of instrument and fed back to control system to remove by calibration. Actually, to relax the requirement of these systematic error is an effective way to decrease the whole system cost. Some of these incurring factors are random error which is difficult to test the concrete value. But, the value can be estimated in a range based on the real conditions, such as slippage, outside disturbance, etc. For these random errors, the effective method is to control or restrain the incurring factors which will influence the motion accuracy.

REFERENCES

[1] Wang Jian-li, Li-zhen, Liu Hai-ning, “Slippage calculation in friction drive design”, Machine Design and Research, 4，P49-50, (1998).

[2] Liu Bo, Wang Jianhua, “Analyses about the force action and the movement accuracy of the transfer of a friction wheel”, Journal of Xi’an institute of technology, 16, 230-233, (1996).

[3] Wang Jianhua, Lao Qicheng, Liu Bo, “CNC gear measure principle, characteristic and key technology”, Tool technology, 30, 41-43, (1996).

[4] Feng Guonan, “Design and analysis of modern sero-system”, China mechine press, (1990).

[5] Zeng Qing-shuang，Qin Jia-chuan, “Analysis on the low-speed function of turntable servo system”, Journal of Chinese Inertial Technology，9, 64-69, (2001).

[6] Cheng Jun, Li Xinzhi, “The design of a coupling for high precision axle-angle position coder”, Modern radar, 3, 76-81, (1997).

[7] Xia Songbo, Zhang Xinjiang, Liu Zhansheng, “Research on the rotating mechanism misalignment failure”, Journal of vibration measurement & diagnosis, 18, 157-161, (1998).

[8] Zhou Zhukui, “Ellipse perimeter approximately formular”, Mathe report, 6, 45-47, (1995).