

# Method of Stressed Lap Shape Control for Large Mirror Fabrication

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## ABSTRACT

NIAOT has made a stressed lap polishing machine and finished a  $\phi$ 910mm, F/2.0 paraboloid. In the process we found shape control strategy is an important technology for stressed lap polishing tool and this kind of content has not been discussed systematically before. So this paper mainly dedicates to the method of lap shape control. Firstly a mathematical model of stressed lap is introduced. Then three shape control methods are put forward one by one concerning aspects as shape accuracy and deformation hysteresis. The fundamental method is least square algorithm. On the base of it we put forward its reformation form: least square algorithm with damping factor. To get more satisfied performance a new algorithm using optimization under constrains of linear inequalities is proposed. Through theoretical analysis and computer simulation some comparisons are made among three methods. Finally we have done experiments using stressed lap polishing machine in NIAOT and the results obtained substantiate the feasibility and efficiency of our method.

**Key words:** Optical fabrication, stressed lap, linear inequalities, aspheric

## 1. INTRODUCTION

Modern astronomical telescope needs high precision, large aperture and deep asphericity mirrors which depend greatly on optical fabrication technology. Among numerous selections, stressed lap is a special one. It uses deformable large tool to polish aspherical mirrors<sup>[1]</sup>. Compared with small tool polishing technique, it shares both the advantage of high polishing rate and natural smoothness over wide spatial frequencies. Since 1998, Nanjing Institute of Astronomical Optics & Technology (NIAOT), Chinese Academy of Science has developed this technique and achieved success. We have finished  $\phi$ 910mm F/2.0 paraboloid mirror to 23ns RMS using stressed lap<sup>[2]</sup>. One main difficulty we met is shape control technology which guarantees an accurate fit between lap and aspherical surface. In this paper, main emphasis is putting on lap surface control algorithms with aspects as precision and dynamic performance. To give the reader a clear concept of how stressed lap works we briefly describe the mechanical structure and test system of stressed lap in Section 2. Then elicit the mathematical model of deformation behind of the complex structures. Section 3 would focus on shape control using least square algorithm. On the base of analyzing the limitations of LS method we import damping factor to improve it. Difficulties of polishing faster focus ratio aspherics will be discussed in Section 4. After that in order to get more satisfied performance we introduce linear inequalities method and demonstrate its application in lap shape control in Section 5. Section 6 will pay attention to the dynamic character of lap which has great influence on both precision and efficiency.

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## 2. BRIEF DESCRIPTION OF LAP DEFORMATION

### 2.1 Mechanical structure and testing system

The deformable part of stressed lap is a circular aluminum plate. As showed in figure 1, there are twelve steel tubes around the plate. Each tube equips with one actuator and one tension sensor. Three actuators compose a group and are arranged to form an equilateral triangle. One end of tension wire is connected to actuator and the other is fastened on another tube's tension sensor. Actuators generate bending and twisting moments around the edge of plate by loose or tighten the tension wire under control of computer.

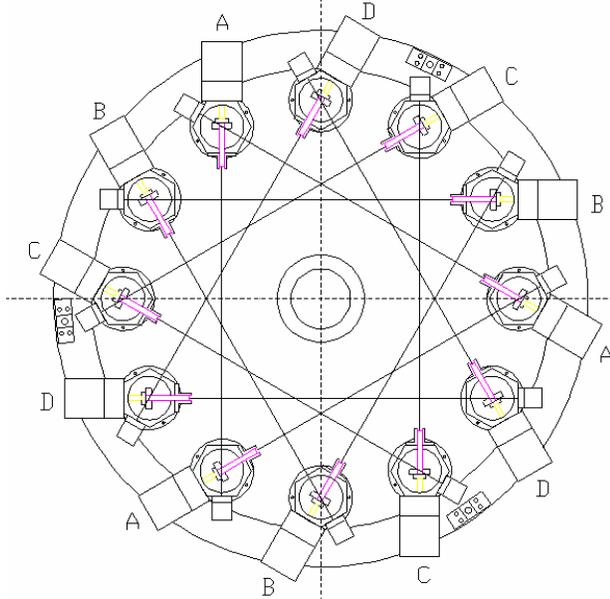


Figure 1 Top schematic of stressed lap

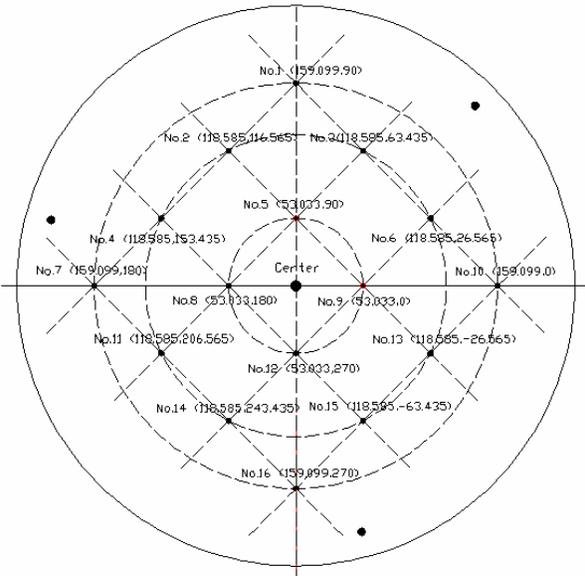


Figure 2 LVDT testing point arrangement

Shape calibration is carried by sixteen LVDT (Linearly Variable Differential Transformer) sensors. They are arranged into square arrays shown in Figure 2. Test point 5, 8, 9, 12 measures the deformation on the inner round of the lap. Other point such as 2, 3, 4, 6, 11, 13, 14 and 15 corresponds to deformation on the middle round. And the left 1, 7, 10, 16 will take charge of outer round deformation. Before formal apply stressed lap to polish mirror, we use LVDT testing platform to adjust the lap to its best state.

### 2.2 Mathematical model for deformation

The specific force of each actuator is determined by two factors. One is the given aspheric shape under the lap. The other is lap's rotation degree. During polishing, lap rotates around the spindle and should change its shape correspondingly. In our system we update surface shape every 5 degree. So when lap finishes one rotation it also completes 72 shape changes. To maintain best fit between the lap plate and mirror in the whole process, the forces should be precisely calculated. Since the deformation is small and aluminum lap always work under elastic limit, we can take granted that stressed lap is an ideal elastomer. Thus we can establish the following linear relationship between deformations and forces.

$$C \times F^i = U^i \quad (i = 1, 2, \dots, 72) \quad (1)$$

In expression (1)  $F^i$  is force vector representing twelve forces in rotation position  $i$ .  $U^i$  is deformation vector representing desired lap deformations on sixteen testing point on rotation position  $i$ .  $C$  is lap's compliance matrix. To elucidate expression (1) more clearly, we rewrite it in detail.

$$C = \begin{bmatrix} \partial u_1 / \partial f_1 & \partial u_1 / \partial f_2 & \cdots & \partial u_1 / \partial f_{12} \\ \partial u_2 / \partial f_1 & \partial u_2 / \partial f_2 & \cdots & \partial u_2 / \partial f_{12} \\ \vdots & \vdots & \ddots & \vdots \\ \partial u_{16} / \partial f_1 & \partial u_{16} / \partial f_2 & \cdots & \partial u_{16} / \partial f_{12} \end{bmatrix} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,12} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,12} \\ \vdots & \vdots & \ddots & \vdots \\ c_{16,1} & c_{16,2} & \cdots & c_{16,12} \end{bmatrix} \quad (2.1)$$

$$F^i = \begin{bmatrix} f_1^i & f_2^i & \cdots & f_{12}^i \end{bmatrix}^T \quad (i = 1, 2, \dots, 72) \quad (2.2)$$

$$U^i = \begin{bmatrix} u_1^i & u_2^i & \cdots & u_{12}^i \end{bmatrix}^T \quad (i = 1, 2, \dots, 72) \quad (2.3)$$

Compliance matrix  $C$  can either be obtained using lap's finite element model or be measured through experiments. If the shape of mirror we polish is determined, the desired deformation vector  $U^i$  is also set. Now the problem is very clear: how to calculate most 'satisfied'  $F^i$ . We shall notice that since the number of test points is larger than that of actuators, there is, in general, no  $F^i$  which can satisfy (1) strictly. So we could only minimize the sum of residual errors squared. The following discussion on force control algorithms is carried under this architecture.

### 3. LAP SHAPE CONTROL USING LEAST SQUARE METHOD AND ITS IMPROVEMENT

#### 3.1 Using least square method to obtain deformation forces

The most manifest solution to above problem is least square (LS) method. Its aim can be expressed as:

$$\text{Min}\{(CF^i - U^i)^T (CF^i - U^i)\} \quad (i = 1 \dots 72) \quad (3.1)$$

And the LS solution is:

$$F^i = (C^T C)^{-1} C^T U^i \quad (i = 1 \dots 72) \quad (3.2)$$

The advantage of LS solution is it provides minimum sum of residual error squared and needs very small amount of calculation.

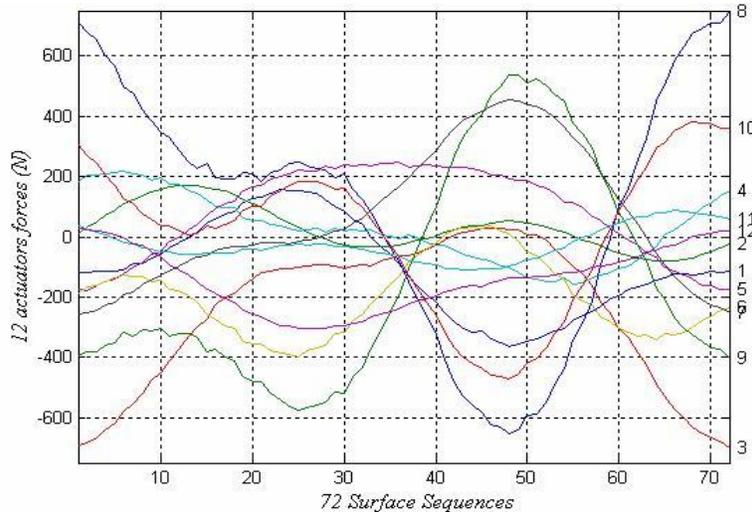


Figure 3 forces diagram of 12 actuators with 72 divisions around a rotation using LS method (F/2.0 paraboloid)

Figure 3 shows the force diagram of 12 actuators using LS method. The 12 lines represent 12 actuators' forces. 72 surface sequences on horizontal axis means lap shape changes every 5 degree. The force range is about 1500 Newton. Due to mechanical and electrical restriction, each actuator in our machine could provide four hundred Newton at most. After this calculation we can see the drawback of LS method very clear: it could not regulate output forces range and in most circumstances force range may exceed the capability of actuator so much that it couldn't be executed to practical stressed lap.

### 3.2 Using damp least square method to obtain deformation forces

So some improvement should be done to constrain forces to an acceptable range meanwhile fit precision should also be guaranteed. Fortunately, the damp least square method (DLS) (Levenberg 1944) can provide such a solution. It has also been used in active optics in LAMOST successfully [5]. The aim of DLS method is:

$$\text{Min}\{(CF^i - U^i)^T (CF^i - U^i) + P(F^{iT} F^i)\} \quad (i = 1 \dots 72) \quad (4.1)$$

And the DLS solution is:

$$F^i = (C^T C + PI)^{-1} C^T U^i \quad (i = 1 \dots 72) \quad (4.2)$$

Here P is a positive number called damp factor and I is a unit matrix. The key to understand DLS method is damp factor P. When a bigger the damp factor is selected in calculation, the force range may reduce a bit but fit residual error may increase a little. Two extreme conditions may help to explain this tendency more clearly. If we set damp factor to zero, DLS method degrade into LS method which has highest precision but greatest force range. Otherwise if damp factor is infinite, force range tends to be zero and we get worst fit precision.

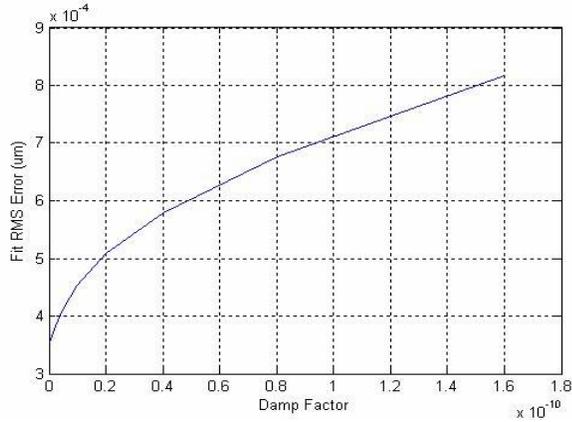


Figure 4 (a) damp factor and force range

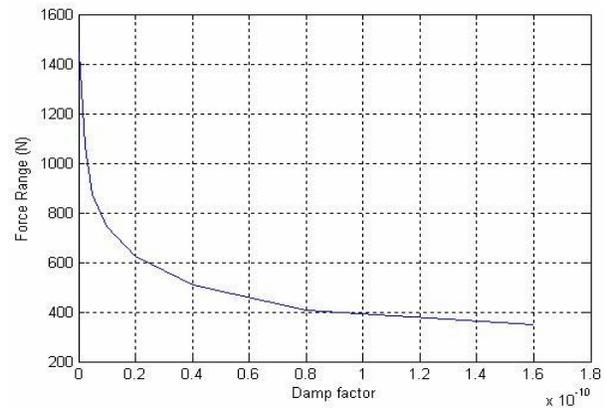


Figure 4 (b) damp factor and fit residual error

Figure 4(a) illustrates the connection between damp factor and force range. Figure 4(b) shows the relation of damp factor and fit residual error. From above analysis the value of damp factor represents a compromise between fit precision and force range. By adjusting it we can control force into a practicable range and obtain an acceptable surface fit precision.

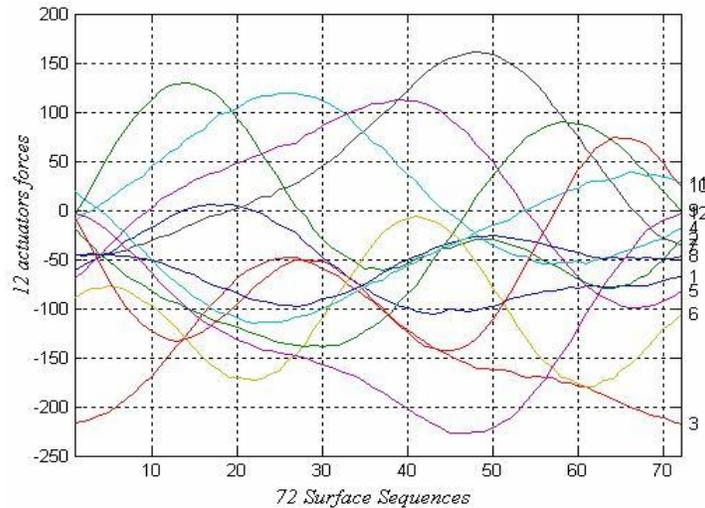


Figure 5 forces diagram of 12 actuators with 72 divisions around a rotation using DLS method (F/2.0 paraboloid)

Figure 5 shows the force diagram of 12 actuators using DLS method. The force range is about 390 Newton and shape accuracy is RMS 0.7um.

#### 4. DIFFICULTIES IN POLISHING FASTER ASPHERICS

Former discussion on lap deformation is mainly based on  $\phi 910\text{mm}$  F/2.0 paraboloid. Can it be used to polish faster aspherics such as F/1.5 or F/1.2 mirror? First of all we should find out the difficulties in polishing faster aspherics.

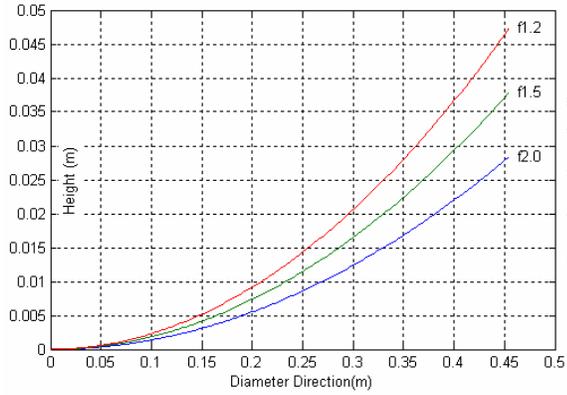


Figure 6(a) Meridian Section of F/2.0, F/1.5 and F/1.2 Paraboloid

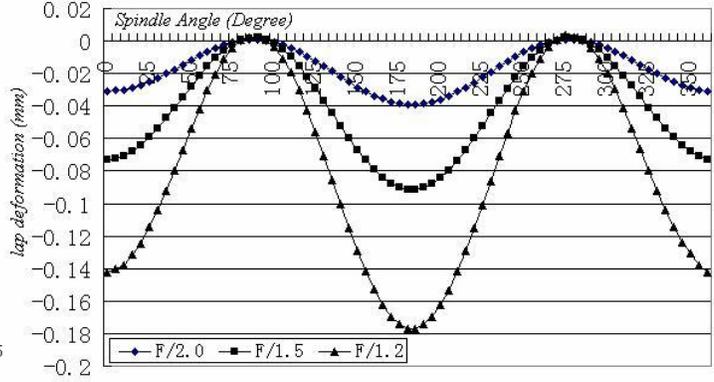


Figure 6(b) Lap Deformation of F/2.0, F/1.5 and F/1.2 Paraboloid (No.7 Test point in Figure 2)

Figure 6(a) shows shapes of F/2.0, F/1.5 and F/1.2 respectively along meridian section. Lap deformation on No.7 test point is picked out in figure 6(b) to explain what will happen when asphericity becomes greater. According to optical calculation, the range of deformation increases with asphericity which may rise dramatically with focal ratio. For instance, the deformation scope of F/2.0 is 0.07mm; F/1.5 turns out to be 0.16mm and F/1.2 goes up to 0.32mm. To satisfy the increasing departure amplitude on lap we need a ‘stronger’ actuator which is able to output greater moment. If lap maintains its rotational velocity as before, the speed of deformation will also come up to a higher level as shown in figure 6(b). This time means a more powerful servo system is needed. Another problem caused faster focal ratio is deformation hysteresis. This may cause by the backlash among mechanical components in actuators, lag of servo system, and inaccuracies of lap’s deformation model. For all these reasons, DLS method could not provide an acceptable solution for F/1.5 and F/1.2 paraboloid.

Table 1 Lap deformation accuracy, force range and damping factor for F/1.5 paraboloid

damping factor	0.0	$2 \times 10^{-10}$	$5 \times 10^{-10}$	$1 \times 10^{-9}$	$2 \times 10^{-9}$	$4 \times 10^{-9}$	$8 \times 10^{-9}$
Residual Error RMS ( $\mu\text{m}$ )	0.83	2.03	2.64	3.12	3.60	4.19	5.09
force range (N)	3357.9	769.0	564.8	439.7	342.8	280.0	232.0

Table 2 Lap deformation accuracy, force range and damping factor for F/1.2 paraboloid

damping factor	0.0	$1 \times 10^{-9}$	$2 \times 10^{-9}$	$4 \times 10^{-9}$	$1 \times 10^{-8}$	$2 \times 10^{-8}$	$4 \times 10^{-8}$
Residual Error RMS ( $\mu\text{m}$ )	1.59	5.98	6.92	8.05	10.57	14.21	20.39
force range (N)	6459.7	845.2	659.1	538.0	421.4	352.3	290.7

According to data in table 1 and table 2, we found that for F/1.5 and F/1.2 paraboloid either the surface residual error exceeds the standard we set or the force range surpasses the ability of actuator. As focal ratio fastens, the contradiction between accuracy and force range becomes more and more obvious. We should find new solution to solve this problem.

## 5. SHAPE CONTROL BY OPTIMIZATION UNDER CONSTRAINTS OF LINEAR INEQUALITIES

At present, there are two possible ways to improve lap's deformation precision. One is to redesign the entire lap: reducing the thickness of plate to make it more flexible or enhancing the actuator and servo system which will supply more distortion moment. The other way depends on bettering our algorithm. Obviously the latter is more economical and convenient. It needs no modification on hardware, so we can test the result on machine just at hand. First of all, hindrance factors in DLS method should be found out. We notice that the damping factor  $P$  in expression (4.2) is multiplied to unit matrix  $I$ . So it has influence on every actuator. And every outputting force is thus constrained. Actually, only some actuators need to be constrained and others should be released. In other words DLS method may over restricts actuators. Secondly, although the force range can be adjusted by damping factor, the information of force range has no direct connection with whole calculation. We just select an appropriate damping factor afterward. So adding more information to computation may also improve the performance.

After these analyses we can put forward our new algorithm using optimization under constraints of linear inequalities (OCLI). It can be expressed as followings:

$$\begin{cases} \text{Min}\{(CF^i - U^i)^T (CF^i - U^i)\} & (i = 1 \cdots 72) & (5.1) \\ f_j^i - f_{\min} > 0 & (j = 1 \cdots 12) & (5.2) \\ f_{\max} - f_j^i > 0 & & (5.3) \end{cases}$$

(5.1) is the target of optimization that is minimizing the residual error. Expression (5.2) and (5.3) describe the constraints for forces:  $f_{\min}$  is the low boundary for distortion force and  $f_{\max}$  is the up limit correspondingly. These two values are both set before calculation. Using linear inequalities in (5.2) and (5.3) we draw the outline of an operative area. Forces in this area can be executed by actuators smoothly. Apparently the force range could be calculated by subtract  $f_{\min}$  from  $f_{\max}$ . So we can control force range directly without difficulty. To solve problem like (5.1~5.3), modern quadratic optimization technology proved to be a powerful weapon. We use interior point method (IPM) to solve it. Unlike LS or DLS method which can be finished without iteration, IPM needs it. Although it may cost more time and computational resources, this proves to be no trouble. In each iterative circle, IPM transfers a constraint optimization problem to an unconstrained one and then solves it. After several iterations it finally finds which actuator should be constrained and which should be released and then result can be given out. We test this method for F/1.5 paraboloid and analyze experiment results.

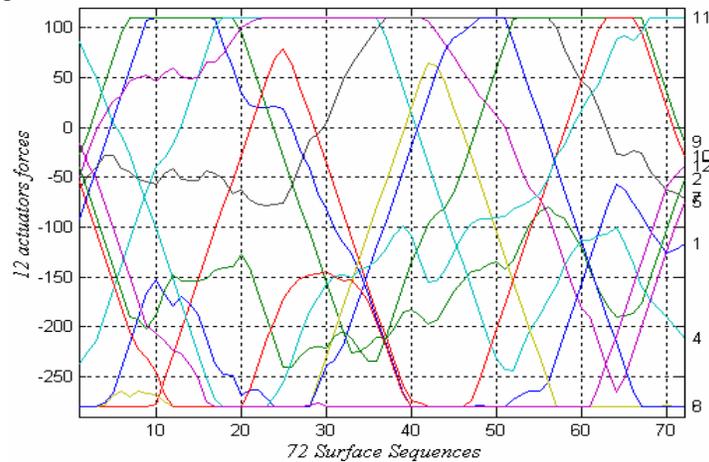


Figure 7 forces diagram of 12 actuators with 72 divisions using OCLI (F/1.5 paraboloid)

Figure 7 is 12 actuators' force diagram using OCLI method under linear inequalities for F/1.5 paraboloid. The force range is about 390 Newton and shape accuracy is RMS 2.52um. By contrast, to get the same force range, using DLS

method the precision will reduce to RMS 3.37 $\mu\text{m}$  ( $P=1.4 \times 10^{-9}$ ). Another comparison can be carried by examining Figure 5 and Figure 7 carefully. In Figure 5 (DLS method) most forces are wandering near the middle part of force range. Seldom is the up boundary touched, neither is the low boundary. However in figure 7, up and low boundary of force become a welcome place. Many forces touch these two lines. Those arrive up boundary is constrained by (5.3) and those reach low boundary is restricted by (5.2). It seems that all forces are evenly distributed in whole of the range. This difference in force distribution shows OCLI method use limited force range more efficiently than DLS method. We listed different force range and surface precision for F/1.5 and F/1.2 paraboloid in table 3 and table 4 correspondingly.

Table 3 Lap deformation accuracy, force range for F/1.5 paraboloid

Residual Error RMS ( $\mu\text{m}$ )	0.83	1.50	1.89	2.28	2.67	3.01	3.34
Pre-set force range (N)	$\infty$	769.0	564.8	439.7	342.8	280.0	232.0

Table 4 Lap deformation accuracy, force range for F/1.2 paraboloid

Residual Error RMS ( $\mu\text{m}$ )	1.59	4.38	5.14	5.78	6.62	7.27	8.03
Pre-set force range (N)	$\infty$	845.2	659.1	538.0	421.4	352.3	290.7

Figure 8 shows the relation between surface residual RMS error and force range for DLS method and OCLI method. The figure substantiates the conclusion that under same force range OCLI method provides a more precise solution than DLS method. And these results have been tested on our  $\phi 450\text{mm}$  lap.

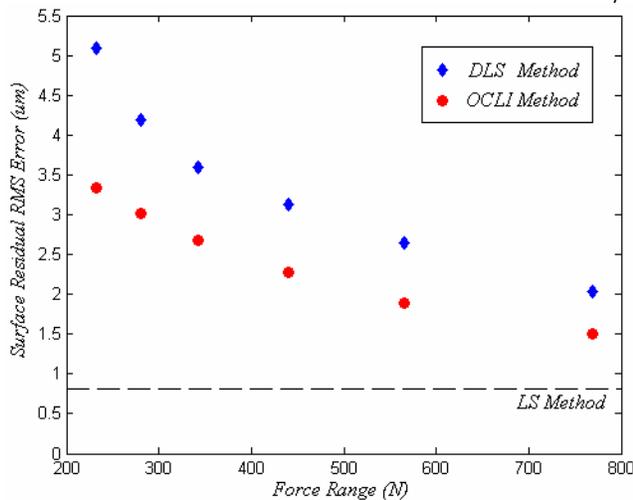
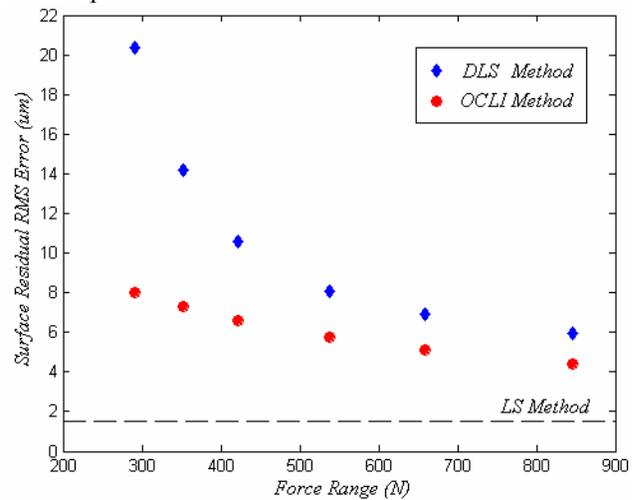


Figure 8(a) Relation of surface residual RMS error and force range for F/1.5 paraboloid (DLS method and OCLI method)



(b) Relation of surface residual RMS error and force range for F/1.2 paraboloid (DLS method and OCLI method)

Basing on experiences we got on polishing mirror, deformation residual RMS error less than 3 $\mu\text{m}$  can be used in practical work. Then combined with analysis upward, our stress lap can polish  $\phi 910\text{mm}$  F/1.5 paraboloid without modification in structure. As to faster aspherics, it would be incompetent.

## 6. DYNAMIC CONSIDERATION FOR LAP

The model we establish for lap in expression (1) is a static one. Unfortunately the essential character of lap deformation is dynamic. As asphericity goes deeper, both the amplitude and the speed of deformation increase which give fertile land to show dynamic character. If the deformation frequency rises, we can observe shape amplitude decreases and phase lags. So it is necessary to study the dynamic model of lap. According to elastic mechanics, the dynamic model of lap can be expressed as following<sup>[7]</sup>:

$$[M]\{\ddot{u}\} + [D]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \quad (6)$$

Where [M] is mass matrix, [D] is damping matrix and [K] is stiffness matrix. The existence of inertial force ( $[M]\{\ddot{u}\}$ ) and damping force ( $[D]\{\dot{u}\}$ ) are the dynamic terms which are eliminated in expression (1). If we use the dynamic model for control, it will definitely improve the performance of lap especially for fast focus ratio aspherics. Yet there are several obstacles to overcome. First is how to obtain accurate parameters in mass matrix and damping matrix. Since the complicated structure of lap, it is difficult to establish precise model using finite element method. If system identification technology is applied to estimate these parameters, the shape calibration needs to be upgrade to a high level. It should be able to record the transient small changes accurately and timely during whole process of deformation. Secondary using dynamic model will lead to the rise of amount of computation. It may be thousand times than that for static model. All these need to be researched carefully before dynamic model is applied formally. But it is worthwhile.

## 7. CONCLUSION

This is the first stressed lap made in NIAOT. Some modifications are carried and a new design is under way. We can expect a more satisfied performance would obtain in near future. At the same time a 2.5m NC optical machine with 5 axes is also under construction. We believe with all these efforts we paid on hardware and software, the optical manufactory ability in NIAOT will upgrade to a higher level.

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## REFERENCES

1. S.C. West, H.M. Martin, R.H. Nagel, etc, "Practical design and performance of the stressed lap polishing tool", *Applied Optics*, Vol. 33, Page 8084-8099, 1994.
2. Cui Xiangqun, Gao Bilie, Wang Daxing etc., "A New Polishing Technology For Large Diameter & Deep Aspherical Mirror", *Acta Optica Sinica*, 2005, 25(3), p402~p407(in Chinese).
3. Wang Daxing, Li Ying, Yang Shihai, Gao Bilie, "Study on Control technology of active stressed lap polishing aspherical mirror", *Optical Technique*, 2005, Vol.31, No.3, p373~p379(in Chinese).
4. Zhu Zheng, Gao Bilie, etc., "Optical Technology and Testing Method Using Stressed Lap", *Optical Technique*, 2005, 31(3), p341~p343 (in Chinese).
5. Su Dingqiang, Cui Xiangqun, "Active Optics in LAMOST", *Chinese journal of Astronomy and Astrophysics*, 2004, Vol.4, No.1, p1~p9.
6. Li Ying, Wang Daxing, "Study on distortion control technology of the active stressed lap polishing deeper aspherical mirror", Proc. SPIE Vol. 6024, p. 450-457, ICO20: Optical Devices and Instruments.
7. Wang Lei, Zhu Yongtian, Zhang Qingfeng, "Mechanical design of the stressed-lap polishing tool", Proc. SPIE Vol. 6024, p. 471-477, ICO20: Optical Devices and Instruments.