

Active Optics in LAMOST

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Abstract Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) is one of the major national projects under construction in China. Active optics is one of the most important technologies for new large telescopes. It is used for correcting telescope errors generated by gravitational and thermal changes. Here, however, we use this technology to realize the configuration of LAMOST, —a task that cannot be done in the traditional way. A comprehensive and intensive research on the active optics used in LAMOST is also reported, including an open-loop control method and an auxiliary closed-loop control method. Another important development is in our pre-calibration method of open-loop control, which is with some new features: simultaneous calculation of the forces and displacements of force actuators and displacement actuators; the profile of mirror can be arbitrary; the mirror surface shape is not expressed by a fitting polynomial, but is derived from the mirror surface shape formula which is highly accurate; a proof is given that the solution of the pre-calibration method is the same as the least squares solution.

Key words: galaxies: distances and redshifts — techniques: active optics — telescopes — instrumentation: adaptive optics

1 INTRODUCTION

After China completed its 2.16 m telescope in the end of 1980's, in the early of 1990's Chinese astronomers discussed the question as to which new facility should be developed in China for the next stage. In astronomy the physical information obtained from optical spectroscopy is unsurpassed in richness and sophistication. However in the early 1990's, the number of celestial objects with recorded slit spectra is only 10^5 , out of a total of 10^9 recorded objects. Wang Shou-Guan (S. G. Wang) suggested that the next facility should be used mainly for extensive astronomical spectroscopic observation. His suggestion won the agreement of most Chinese astronomers. In 1994, Wang Shou-Guan and Su Ding-Qiang (D. Q. Su) put forward the initial configuration of the Large Sky Area Multi-Object Fiber Spectroscopic Telescope (LAMOST) (Wang et al. 1994; Wang et al. 1996; Wang 2003). In 1997, The National Plan Committee of China approved this project. It is now one of the major national projects under construction. Recent status of LAMOST has been reported in (Su & Cui 2002).

LAMOST is a special reflecting Schmidt telescope with a 4m aperture and a 5° field of view (FOV). It has a focal length of 20m and an f-ratio of 5. It lies on the ground. Its optical axis is

fixed in the meridian plane and is tilted by 25° to the horizontal (see Fig. 1 and Fig. 2). Celestial objects are observed for 1.5 hours as they cross the meridian. During the observation only the alt-azimuth mounting of the reflecting Schmidt correcting mirror (Ma) and the focal surface do the tracking. The sky area to be observed is $-10^\circ \leq \delta \leq +90^\circ$. Both the correcting mirror Ma and the spherical mirror Mb are segmented mirrors, consisting of 24 and 37 sub-mirrors respectively. The main work of LAMOST is multi-object fiber spectroscopic observation. 4000 optical fibers will be put on the focal surface. These optical fibers are linked to 16 low and medium dispersion spectrographs. The fibers have a diameter of 3.3 arcsec. The image quality required is that 80% of the optical energy should be within 1.5 arcsec. Some of the projects planned for LAMOST are: 1) A galaxy redshift survey reaching to 20.5^m ; 2) Identification of numerous objects found by X-ray, IR and radio surveys; 3) Spectroscopic follow-up observation for many special celestial objects based on multi-color photometry; 4) Variable objects observed repeatedly.

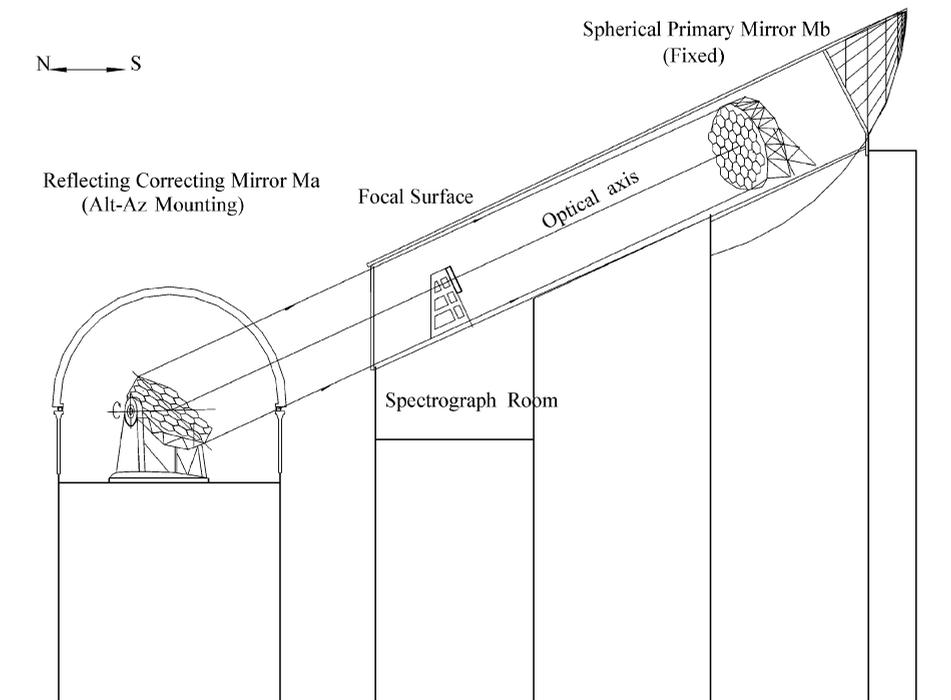


Fig. 1 LAMOST: a general view.

2 THE BASIC ACTIVE OPTICS METHOD IN LAMOST

2.1 The Shape of the Reflecting Correcting Surface of Ma and the Core Idea of Active Optics in LAMOST

See Fig. 1 and Fig. 2, O_0 is the spherical center of the spherical mirror Mb. The optical axis $O_0 B$ passes through O_0 , lies in the meridian plane and is inclined at 25° to the horizontal. In a Schmidt system, the center (vertex) of the reflecting correcting surface of Ma should be at O_0 and the surface center of Mb should be on the optical axis. After reflecting by Ma, the ray

SO_0 which arriving O_0 and from the center celestial object of FOV, will be along the optical axis O_0B . The incident plane is the plane that includes the optical axis and the ray SO_0 . O_0N is the angle bisector of the angle between the optical axis O_0B and the ray SO_0 . Plane A is the plane normal to O_0N through O_0 . It is apparent that plane A is also the tangential plane at the vertex of the reflecting correcting surface of Ma. In plane A we create two coordinate systems (see Fig. 3): $O_0y_0z_0$ and $O_0y_1z_1$, with O_0y_0 along the section of the vertical plane, and O_0y_1 along the section of the incident plane.

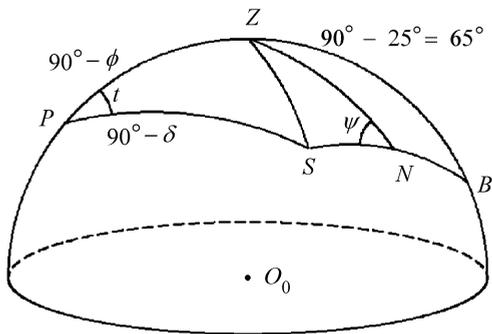


Fig. 2 Basic points and lines in the celestial sphere.

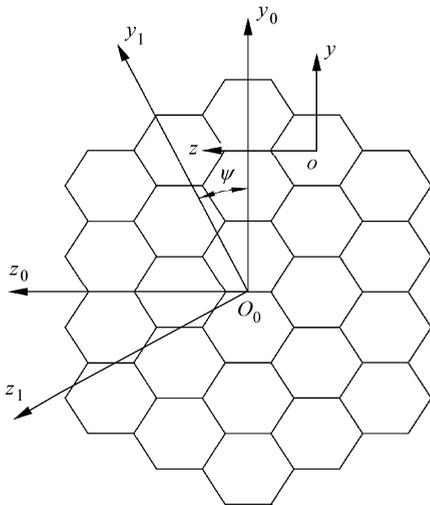


Fig. 3 Three coordinate systems in relation to Mirror Ma.

From eliminating the third-order spherical aberration the shape of reflecting correcting surface of Ma can be obtained (Wang et al. 1996):

$$s = -\frac{1}{64f^3 \cos \theta} \left[(y_1^2 \cos^2 \theta + z_1^2)^2 - 2kr^2 (y_1^2 \cos^2 \theta + z_1^2) \right], \quad (1)$$

where s is the displacement from plane A to this surface at point (y_1, z_1) , (a right hand coordinate system consists of the positive direction of s , O_0y_1 -axis and O_0z_1 -axis), f is the focal length, θ is the incident angle of the ray SO_0 , r is the radius of the clear aperture, and $k = r_0^2/r^2$, where r_0 is the height of the neutral zone. In a reflecting Schmidt system, the off-axis image spread is at a minimum when $k = r_0^2/r^2 = 1.5$ (Lemaitre 1983). Although formula (1) is derived by eliminating only the third-order spherical aberration, it is highly accurate for such a reflecting Schmidt telescope as LAMOST.

Ma is mounted on an alt-azimuth mounting. O_0N is its normal at its vertex. So the directions of O_0y_0 -axis and O_0z_0 -axis are fixed with respect to the profile of Ma.

For a given (t, δ) of the object at the center of the FOV, spherical trigonometry formulae can be used to obtain the incident angle θ and the rotating angle ψ from the O_0y_0 -axis to the O_0y_1 -axis (see Appendix). Then formula (1) gives the shape s of the reflecting correcting surface Ma. For different objects, (t, δ) are different, and so is the surface shape s . Even for the same celestial object, the hour angle t changes during its observation, and the surface shape s also varies. Such an optical system with continuously varying shape cannot be realized by traditional means. Active optics is one of the most important technologies for building large

telescopes, it is used for correcting telescope errors generated by gravitational and thermal changes (Wilson et al. 1987; Schneermann & Cui 1988; Noethe et al. 1988; Wilson et al. 1989; Wilson et al. 1991). This technology has made the new generation monolithic thin-mirror large telescopes realizable with excellent image quality and at an acceptable cost. However, here, we put forward the idea of using this technology to realize an optical system that could not be realized in the traditional way (Su et al. 1986). LAMOST is based on such an application of active optics (Wang et al. 1994; Wang et al. 1996). That is, here, we use active optics method to change the shape of Ma in realtime and so realize this particular reflecting Schmidt system.

2.2 Basic Formula for Obtaining the Forces and Displacements to Create the Surface Shape of the Sub-mirror of Ma

Each sub-mirror of Ma is a hexagonal with 1.1 m along the diagonal and 25 mm thick. The original shape of the reflecting surface is plane. At the back of each sub-mirror (Fig. 4) there are 34 simple force actuators each with a force sensor (load sensor) and 3 displacement actuators (changeable fixed points). All the actuators are used for obtaining the required shape of Ma. The maximum required deformation of the surface shape of the sub-mirrors is about $10\mu\text{m}$. During observation, the maximum time interval for adjusting Ma is two minutes. Each sub-mirror of Mb is a hexagon of the same size (1.1 m along the diagonal) but a thickness at 75 mm. Its surface shape is spherical and is fixed. There are only three displacement actuators (changeable fixed points) at the back of each sub-mirror of Mb. These displacement actuators are only used for correcting the thermal deformation of the support system of Mb.

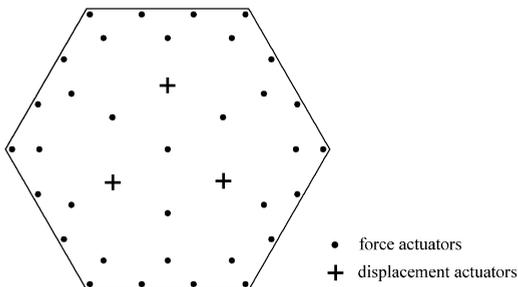


Fig. 4 A sub-mirror of Ma.

For a given sub-mirror of Ma, we create an oyz coordinate system, again in the plane A. The origin o of oyz is the projection of the center of the given hexagonal sub-mirror on the plane A and the oy and oz axes are respectively parallel to the O_0y_0 and O_0z_0 -axes. The directions of oy -axis and oz -axis are fixed relative to the profile of the given sub-mirror. For a given (t, δ) of the central object we can calculate the incident angle θ (see Appendix). Choosing $k = 1.5$. Then all coefficients in (1) are known. The coordinates of o in the $O_0y_0z_0$ coordinate system are known. The rotating angle ψ can also be calculated

(see Appendix). Then the transfer formulae from the oyz to the $O_0y_1z_1$ coordinate systems are obtained. Put these into (1) we obtain

$$s = a_1y^4 + a_2y^3z + a_3y^2z^2 + a_4yz^3 + a_5z^4 + a_6y^3 + a_7y^2z + a_8yz^2 + a_9z^3 + a_{10}y^2 + a_{11}yz + a_{12}z^2 + a_{13}y + a_{14}z + a_{15}. \quad (2)$$

All the coefficients a_1, a_2, \dots, a_{15} are known.

Let \mathbf{l} be the vector whose components are the forces of the force actuators and the displacements of the displacement actuators for a given sub-mirror, and n be the number of its components (for LAMOST $n = 34 + 3 = 37$). Let \mathbf{s}' be the vector whose components are the shape values at m points on the surface of the sub-mirror,

$$\mathbf{l}(f_1, f_2, \dots, f_{34}, d_1, d_2, d_3), \\ \mathbf{s}'(s'_1, s'_2, \dots, s'_m).$$

We take \mathbf{l} as the variable and \mathbf{s}' as the function of \mathbf{l} . The following matrix C can be calculated from mechanics (for the 34 force columns) and from simple geometry (for the three displacement columns):

$$C = \begin{bmatrix} \frac{\partial s'_1}{\partial f_1} & \frac{\partial s'_1}{\partial f_2} & \dots & \frac{\partial s'_1}{\partial f_{34}} & \frac{\partial s'_1}{\partial d_1} & \frac{\partial s'_1}{\partial d_2} & \frac{\partial s'_1}{\partial d_3} \\ \frac{\partial s'_2}{\partial f_1} & \frac{\partial s'_2}{\partial f_2} & \dots & \frac{\partial s'_2}{\partial f_{34}} & \frac{\partial s'_2}{\partial d_1} & \frac{\partial s'_2}{\partial d_2} & \frac{\partial s'_2}{\partial d_3} \\ \frac{\partial s'_m}{\partial f_1} & \frac{\partial s'_m}{\partial f_2} & \dots & \frac{\partial s'_m}{\partial f_{34}} & \frac{\partial s'_m}{\partial d_1} & \frac{\partial s'_m}{\partial d_2} & \frac{\partial s'_m}{\partial d_3} \end{bmatrix}. \quad (3)$$

From differential calculus, we have the relationship between $d\mathbf{s}'$ and $d\mathbf{l}$,

$$d\mathbf{s}' = C d\mathbf{l}. \quad (4)$$

Under the elastic limit, glass is an ideal elastomer. For an ideal elastomer the components of \mathbf{s}' are strictly linear functions of the components of \mathbf{l} , so (4) should be strictly accurate for any \mathbf{l} under the elastic limit

$$\mathbf{s}' = C\mathbf{l}. \quad (5)$$

We now put $\mathbf{s}(s_1, s_2, \dots, s_m)$, in which s_1, s_2, \dots, s_m are the shape values at m points equally distributed on the reflecting correcting surface of the sub-mirror to be calculated from (2), instead of the \mathbf{s}' into (5):

$$\mathbf{s} = C\mathbf{l}. \quad (6)$$

Since we have $m \gg n$ (we take $m = 1000$ or more), there is, in general, no \mathbf{l} which can satisfy (6). So we use the method of least squares to obtain \mathbf{l} , by minimizing the sum of residual errors squared. The solution is

$$\mathbf{l} = (C^T C)^{-1} C^T \mathbf{s}. \quad (7)$$

2.2.1 Using the Damp Least Squares Method to Obtain the Forces and Displacements

In the matrix C the force columns are closely correlated. For example, for three columns corresponding to three adjacent force actuators, the middle one will be nearly equal to the mean of the other two. In such a situation if C has some errors, the solution $|\mathbf{l}|$ by the least squares method may be very large. Applying such an \mathbf{l} to the sub-mirror would not be safe, and may even lead to a shape worse than the original. In our first thin-mirror active optics experiment (Su et al. 1994), where the matrix C was obtained by actual measurements, we met this problem; we then used the damp least squares method (Levenberg 1944) and solved the problem successfully. The solution \mathbf{l} of the damp least squares method is

$$\mathbf{l} = (C^T C + P I)^{-1} C^T \mathbf{s}. \quad (8)$$

Here P is a positive number called damp factor and I is the unit matrix. We were afraid that by using either calculations or actual measurements we may not be able to obtain sufficiently accurate C . If so, we will use the damp least squares method again. After such a solution \mathbf{l} is obtained with a suitable damp factor and after applying such forces and displacements to the sub-mirror, the wavefront should be measured. The common instrument for this measurement

is the Shack-Hartmann (S-H) device. In general, the wavefront is not fine enough. Then we should use the damp least squares method again to make a second time correction. It means that we may have to repeat the process several times (it was 2–3 times in (Su et al. 1994)). The sum of the \mathbf{l} , summed over all the iterations, should be the right one we need, and we store it in the computer. We should do this work for each sub-mirror and for many different sky directions (t, δ) , and collect all the results into a look-up table in the computer for the open-loop control (Wang et al. 1996). Making such a look-up table is an extremely complex process, and the accuracy is limited by the S-H measurement.

2.2.2 Using the Least Squares Method to Obtain the Forces and Displacements

Under the elastic limit glass is an ideal elastomer. The ideal elastomer obeys accurate physical rules, for example, the equilibrium equation (Lamé equation)

$$\begin{aligned} (\lambda + \mu) \frac{\partial \theta}{\partial x} + \mu \nabla^2 \xi + \rho X &= 0, \\ (\lambda + \mu) \frac{\partial \theta}{\partial y} + \mu \nabla^2 \eta + \rho Y &= 0, \\ (\lambda + \mu) \frac{\partial \theta}{\partial z} + \mu \nabla^2 \zeta + \rho Z &= 0. \end{aligned} \quad (9)$$

It means that if all the input parameters such as the structure of the given sub-mirror, the positions of all the actuators, the Young's modulus and the Poisson's ratio are accurately known, then we can obtain the matrix C as accurately as is needed. It is more convenient to use the finite element method with a fine enough grid to obtain an accurate C than solving the Lamé equation. For such an accurate matrix C , we can calculate $(C^T C)^{-1} C^T$ and store it in the computer. For a definite central object (t, δ) , it is convenient and no problem to use Formula (7) to obtain the forces and displacements.

2.2.3 Using the Pre-calibration Method to Obtain the Forces and Displacements

Define the vectors

$$\begin{aligned} \mathbf{s}_1 & (y^4|_1, \quad y^4|_2, \quad \cdots, \quad y^4|_m) \\ \mathbf{s}_2 & (y^3 z|_1, \quad y^3 z|_2, \quad \cdots, \quad y^3 z|_m) \\ & \vdots \\ \mathbf{s}_{14} & (z|_1, \quad z|_2, \quad \cdots, \quad z|_m) \\ \mathbf{s}_{15} & (1, \quad 1, \quad \cdots, \quad 1). \end{aligned}$$

It is clear that

$$\mathbf{s} = a_1 \mathbf{s}_1 + a_2 \mathbf{s}_2 + \cdots + a_{14} \mathbf{s}_{14} + a_{15} \mathbf{s}_{15}. \quad (10)$$

Put (10) into (7), we obtain

$$\mathbf{l} = a_1 (C^T C)^{-1} C^T \mathbf{s}_1 + a_2 (C^T C)^{-1} C^T \mathbf{s}_2 + \cdots + a_{14} (C^T C)^{-1} C^T \mathbf{s}_{14} + a_{15} (C^T C)^{-1} C^T \mathbf{s}_{15}. \quad (11)$$

We calculate $(C^T C)^{-1} C^T \mathbf{s}_1, (C^T C)^{-1} C^T \mathbf{s}_2, \cdots, (C^T C)^{-1} C^T \mathbf{s}_{14}, (C^T C)^{-1} C^T \mathbf{s}_{15}$ beforehand and writing them as $\mathbf{l}_1, \mathbf{l}_2, \cdots, \mathbf{l}_{14}, \mathbf{l}_{15}$ and store them in the computer. It is clear that \mathbf{l}_1 is the least squares solution for $\mathbf{s}_1, \mathbf{l}_2$ is the least squares solution for $\mathbf{s}_2, \cdots, \mathbf{l}_{15}$ is the least squares solution for \mathbf{s}_{15} . It is also clear that the vector \mathbf{l}_{15} is $(0, 0, \cdots, 0, 1, 1, 1)$. Formula (11) can be written as

$$\mathbf{l} = a_1 \mathbf{l}_1 + a_2 \mathbf{l}_2 + \cdots + a_{15} \mathbf{l}_{15}. \quad (12)$$

Giving (t, δ) we can obtain all coefficients a_1, a_2, \dots, a_{15} , then by using Formula (12) all the forces and displacements for the sub-mirror are obtained. This is the pre-calibration method for LAMOST. The pre-calibration method has been used for some other large telescopes (Wilson et al. 1987; Schneermann & Cui 1988; Noethe et al. 1988; Wilson et al. 1989; Wilson et al. 1991). However there are some important differences between their cases and ours. 1) All their active mirrors are circular and their formula is suitable only for circular mirrors. Our sub-mirrors are hexagonal and our method can be applicable to any profile mirrors. 2) Their pre-calibration method is only for obtaining the forces, whereas our method includes also the displacements, and we obtain the forces and displacements at the same time; 3) In our method the mirror surface shape is not expressed by a fitting polynomial, but it is derived from the mirror surface shape formula which is with highly accurate; 4) A proof is given that the solution of the pre-calibration method is the same as the least squares solution. Since using (12) will reduce the calculation much more than using (7), we will use (12) in LAMOST.

It should be pointed out that since the gravitational and thermal deformations of the Ma support system are rather large, the displacements of the displacement actuators obtained from (7) or (12) have no absolute meaning, but later we will use the difference of them.

Formula (7) or (12) is an open-loop control method for obtaining the surface shape of the sub-mirrors of Ma.

Some relevant issues should be mentioned at this point: (1) In LAMOST the weight of the sub-mirror is supported by all its force actuators and its three displacement actuators. It is regular and the correcting forces can be calculated according to the zenith distance and applied to all the force actuators. (2) Some other regular correcting forces could also be added by the force actuators, such as the forces for correcting the figuring error of the sub-mirror. (3) If the structure, the positions of the force and displacement actuators, the Young's modulus and the Poisson's ratio vary somewhat from sub-mirror to sub-mirror, we can use different matrices C for different sub-mirrors. It is a basic condition for using open-loop control method that the random (uncertain) errors caused by such forces as friction forces, stick forces and some complex forces coming from the lateral support system etc., must be limited to absolute minimum, that the image degradation due to these errors should be less than one arcsec.

An outdoor active optics experiment system is under construction, which will be described in the paper by X. Q. Cui, D. Q. Su, G. P. Li et al., submitted to the SPIE conference on Large Telescope and Instrumentation 2004. One of its most important objectives is to find which is the right method to obtain the forces of the force actuators in LAMOST, i.e., the least squares method (to use (7) or (12)) or the damp least squares method (for obtaining a look-up table). Although such an experiment has not yet been done, we believe that the least squares method should be used. In spite of this, such experiment system is still important. There are many other experiments that will be done on it: measuring the effects of random (uncertain) forces and finding means to decrease them; testing the force actuators and displacement actuators and their connections to the sub-mirror; testing the lateral support system, and so on.

2.3 The Co-focus of Mb and Ma

Since the required image quality is 80% of the optical energy within 1.5 arcsec only co-focus is needed for all the sub-mirrors of Ma and Mb. This issue has been solved successfully in our segmented-mirror active optics experiment system (Su et al. 1998). There are three displacement actuators (changeable fixed points) at the back of each sub-mirror. These displacement actuators are used to make the sub-mirrors co-focus. The co-focus is measured by a simple

Shack-Hartmann (S-H) device. If this S-H device only collects the light from one sub-aperture of each sub-mirror, then it is not certain that the common intersection of the lights collected is also the intersection of the lights from other sub-apertures (if exist) of the sub-mirrors. If there are at least two sub-apertures in a sub-mirror, then it is sure that the common intersection of these sub-apertures' lights is also the intersection of more sub-apertures' lights. For increasing accuracy, it is better to use more sub-apertures, for example three on each sub-mirror (Su et al. 1998). There is a S-H device at the curvature center of Mb. Before each observation we use it and all displacement actuators of Mb to make all sub-mirrors of Mb co-spherical center. There is another S-H device at the focus of LAMOST. By observing a star with this last S-H device and using all displacement actuators of Ma to make Ma co-focus. Since there is no gravitational change on Mb and during one observation the maximum change in the altitude of Ma is only 0.34° (for 1.5 hours observation) (Su & Wang 1997) the gravitational change is very small, we do not need to make the active correction for gravitational deformation. Because thermal change is slow, it may be enough to adjust the co-focus of Mb and Ma before each observation. During observation the displacements of the displacement actuators of Ma are applied for the difference from their starting position according to (12). Because measuring and adjusting for co-focus are easy (the S-H device only needs to include a few sub-apertures for each sub-mirror and there is no need for strict positioning of the sub-apertures), there is no difficulty even carrying out the operation during observation.

3 THE AUXILIARY CLOSED-LOOP CONTROL METHOD IN LAMOST

We will do our best to reduce the random errors and only use the open-loop control method in LAMOST. But it seems a little risky if we depend completely on the open-loop control method, so we also consider the following closed-loop control method as an insurance and an auxiliary method.

After adding the open-loop control forces and carrying out the co-focus operation we use an S-H device to test the image quality. If the image diameter is larger than 1.5 arcsec, we calculate the shape \mathbf{s} for each sub-mirror according to the S-H test data, and make the corrections. In this situation the calculated shape \mathbf{s} includes measuring errors. Write the calculated shape as $\mathbf{s} + \Delta\mathbf{s}$, where \mathbf{s} is the true shape that should be corrected, and $\Delta\mathbf{s}$ is the measuring error. Put $-(\mathbf{s} + \Delta\mathbf{s})$ as \mathbf{s} in Formula (7), the correcting forces and displacements can be obtained. After adding these forces and displacements the shape of the sub-mirror becomes $\mathbf{s} - (\mathbf{s} + \Delta\mathbf{s}) = -\Delta\mathbf{s}$ (here we neglect the residual error in the least squares method), and this completes the closed-loop correction. Since the matrix C is highly accurate here, we need not use the damp least squares method and only need to do the correction once. However, in this closed-loop correction we should do once S-H measurement.

It should be pointed out that when observing celestial objects near the north pole the projected area of Ma is only about one-half of the area when observing at $\delta = -10^\circ$, and many sub-apertures will fall onto the gaps of Ma and Mb. If we require each sub-mirror to have at least 10 sub-apertures, then the S-H device should include about 1000 sub-apertures (only a rough estimate). In this case we can only obtain about 5–10 coefficients for the fitting polynomial for the shape of each sub-mirror, which means that we can only include third-order astigmatism and a few other lowest space frequency terms. Such a closed-loop correction is complex and its correcting power is limited, and its accuracy is limited by the S-H measurement. By the way, we point out that in this situation such a S-H device can be also used for co-focus of Ma.

APPENDIX Derivation of θ and ψ from (t, δ)

In Fig. 2, P is north celestial pole, Z is the zenith, S is the central object in the FOV, O_o is the spherical center of the spherical mirror Mb, O_oB is the optical axis, ϕ is the latitude of the observer.

From spherical triangle PBS , we have

$$\begin{aligned}\cos BS &= \cos(90^\circ - \phi + 65^\circ) \cos(90^\circ - \delta) + \sin(90^\circ - \phi + 65^\circ) \sin(90^\circ - \delta) \cos t \\ &= \sin(\phi - 65^\circ) \sin \delta + \cos(65^\circ - \phi) \cos \delta \cos t, \\ \theta &= BS/2, \\ \sin \angle ZBS &= \sin t \cos \delta / \sin BS.\end{aligned}$$

From spherical triangle ZBN , we have

$$\begin{aligned}\cos ZN &= \cos \theta \cos 65^\circ + \sin \theta \sin 65^\circ \cos \angle ZBS, \\ \sin \angle BNZ &= \sin \angle ZBS \sin 65^\circ / \sin ZN, \\ \psi &= 180^\circ - \angle BNZ.\end{aligned}$$

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References

- Lemaitre G., 1983, In: M. Capaccioli, ed., IAU Colloq. 78, Astronomy with Schmidt-type Telescopes, Dordrecht: Kluwer, 533
- Levenberg K., Quarterly of Applied Mathematics, 1944, II, 164
- Noethe L., Franza F., Giordano P. et al., 1988, J. Mod. Opt., 35, 1427
- Schneermann M., Cui X. Q., 1988, In: M.-H. Ulrich, ed., ESO Conf. Proc. No.30, Very Large Telescopes and Their Instrumentation, Garching: ESO, 389
- Su D. Q., Cao C. X., Liang M., 1986, In: L. D. Barr ed., Proc. SPIE 628, Conference on Advanced Technology Optical Telescopes III, 498
- Su D. Q., Jiang S. T., Zou W. Y. et al., 1994, In: L. M. Stepp, ed., Proc. SPIE 2199, Conference on Advanced Technology Optical Telescopes V, 609
- Su D. Q., Wang Y. N., 1997, Acta Astrophys. Sin., 17, 315
- Su D. Q., Zou W. Y., Zhang Z. C. et al., 1998, In: P. Dierickx, ed., Proc. SPIE 4003, Conference on Optical Design, Materials, Fabrication, and Maintenance, 417
- Su H. J., Cui X. Q., 2002, In: J. M. Oschmann, L. M. Stepp, eds., Proc. SPIE 4837, Conference on Large Ground-based Telescopes, 26
- Wang S. G., 2003, Chin. J. Astron. Astrophys., 3, 1
- Wang S. G., Su D. Q., Hu Q. Q., 1994, In: L. M. Stepp, ed., Proc. SPIE 2199, Conference on Advanced Technology Optical Telescopes V, 341
- Wang S. G., Su D. Q., Chu Y. Q., Cui X. Q., Wang Y. N., 1996, Appl. Opt., 35, 5155
- Wilson R. N., Franza F., Noethe L., 1987, J. Mod. Opt., 34, 485
- Wilson R. N., Franza F., Giordano P., Noethe L., Tarengi M., 1989, J. Mod. Opt., 36, 1415
- Wilson R. N., Franza F., Noethe L., Andreoni G., 1991, J. Mod. Opt. 38, 219